LPL Tutorial

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Abstract

This paper contains three parts

1. A set of tutor models, models that guide you through several aspects of LPL modeling from a simple production model to more complex once. Successively, other features are shown in the models to illustrate the syntax of the language LPL. By aware that some models need Excel or a database to be installed before running correctly. Otherwise all can be run on the Internet without installing LPL locally.
2. A set of "learn" examples which are model fragments explaining various aspects or functions of the LPL language, they are also linked in the reference manual to explain various concepts of the language.
3. Some drawing examples explaining the drawing library of LPL. LPL contains a library that can generate SVG graphs. They can be interpreted by any browser directly.

This text was automatically generated using LPL's own documentation facility, and illustrates itself an important feature of LPL, namely the "model documentation tool". Going through the examples together with loading and running them using *lplw.exe* or the Internet interface gives you a first overview of the capacity of the modeling system LPL. The reader is asked not only to read this tutorial but also to load and run all models. It is the most efficient way to go into the modeling language. Enjoy it!

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1 A Simple Production Model (tutor01)

—- Run LPL Code , HTML Document –

Problem: [[This part of the documentation is intended to briefly explain the problem.]] A firm produces two type of robots called *Marie* and *Jules*. Three production steps must be carried out:

1. Production of the components: It takes 5 hours for each robot *Marie* and *Jules*, with a total capacity of 350 hours per week,
2. Mounting (capacity = 480): It takes 4 hours for mounting one robot of type *Marie* and 8 hours for one robot of type *Jules*,
3. Testing (capacity = 300): It taks 6 hours for one *Marie* and 2 hours for one *Jules*.

The revenue for *Marie* is \$300 and for *Jules* \$200. There are already 20 robots of type *Marie* and 15 *Jules* ordered. How many robots of each type can be produced per week, if the firm wants to maximize the selling revenue (costs are not considered in this simple problem)? ([2]).

Modeling Steps

[[This part of the documentation explains how the problem can be translated into a mathematical model.]]

1. Let's introduce two variables: x and y for the quantity (per week) of the two types of robots to be produced.
2. The components-step has a capacity of 350 hours per week. A unit of robot *Marie* takes 5 hours, a robot *Jules* also takes 5 hours. Hence we have (the component capacity per week):
$$5x + 5y \leq 350$$
3. The mounting-step has a capacity of 480 hours per week. A robot *Marie* takes 4 hours, a robot *Jules* takes 8 hours. Therefore:
$$4x + 8y \leq 480$$
4. The testing-step has a capacity of 300 hours per week. A robot *Marie* takes 6 hours, a robot *Jules* takes 2 hours. Therefore:
$$6x + 2y \leq 300$$
5. At least 20 of type *Marie* and 15 of type *Jules* must be produced, hence:
$$x \geq 20, \quad \text{and} \quad x \geq 15$$
6. Maximizing the revenue is :
$$\max \quad 300x + 200y$$

The complete problem to solve is then as follows:

$$\begin{array}{ll} \text{max} & 300x + 200y \\ \text{subject to} & 5x + 5y \leq 350 \\ & 4x + 8y \leq 300 \\ & 6x + 2y \leq 300 \\ & x \geq 20 \quad y \geq 15 \end{array}$$

Further Comments: In a graphical way, the model represents the solution space as shown in Figure 1. That is, every (x, y)-point within the gray polygon represents a possible (feasible) production that fulfills all constraints. All other points in the two-dimensional space violate at least one of the constraints.

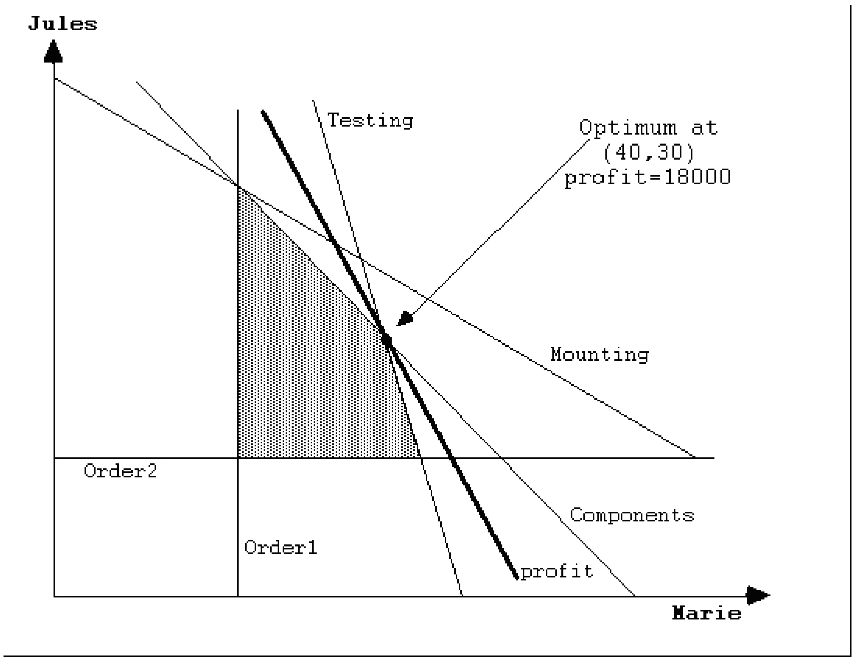


Figure 1: The Feasible Space (grey)

Listing 1: The Complete Model implemented in LPL [2]

```
model TUTOR01 "A Simple Production Model";
  variable  x; y;
  constraint
    Component:  5*x + 5*y  <=  350;
    Mounting:   4*x + 8*y  <=  480;
    Testing:    6*x + 2*y  <=  300;
    Order1:       x        >=   20;
    Order2:             y  >=   15;
  maximize revenue:  300*x + 200*y;
  Writep(x,y);
end
```

LPL Modeling Steps

[[The goal of this tutor is mainly to explain the syntax of LPL. It is done in this text block.]] A LPL coded mathematical model [2] is very close to the usual mathematical notation. However, we have some additional elements to code:

1. Each model coded in LPL begins with a keyword `model` and ends with `end`. (Keywords are all in lowercase.)
2. The code consists of a sequence of *entity* declarations and statements.
3. Each entity declaration begins with a keyword and ends with a semicolon.
4. There are four kinds of entities in this model: `model`, `variable`, `constraint`, and `maximize`. There is no need to repeat the keywords `variable` or `constraint` for consecutive entities of the same type. Therefore `variable` introduces a list of two variables.
5. `constraint` introduces the model constraints. Each constraint then begins with a name. Then follows a colon and the constraint expression.
6. The objective function, with the name `revenue`, begins with `maximize`.
7. Finally, we want to output the solution with the function `Writep`.

Solution: [[This part of the documentation is intended to give a short comment about the solution of the model.]] The optimal solution is $x = 40$ and $y = 30$, which can be verified by Figure 1. It means that neither the quantity of x nor the quantity of y can be increased without violating one of the capacity constraint. (Well, we could augement the quantity of one variable at the expense of the other quantity in this case, but the overall revenue will *always* be smaller.) On the other hand, a reduction of one of the quantities is “suboptimal”: it reduces the `revenue`, and hence it is no longer maximized.

Questions

1. Verify the solution of this model by running it. (Click the link `TUTOR01` at the top of this model description to run the model in the Internet.)
2. What happens if we extend the `Mounting` capacity from 480 to 500?
3. We *must* produce 45 unit of *Jules* instead of 15. How must the model be changed?
4. Besides the three production steps, there is a fourth task: “storage”. The company has a storage capacity of 300 storage units and each Marie uses 7 and each Jules uses 2 storage units. Add this requirements to the model and solve the problem again. Does the new task influence the result?

Answers

1. Locally, run `lplw.exe`. Choose Menu “File/Open” and open the file `tutor01.lpl`. Now choose Menu "Run/Run Model". Click the tab ’TABLE’, and then click on the red node ’x’ in the left part in the LPL application. The value 40 is displayed. Now click on the node ’y’, the value 30 is displayed. OR: Click the red link called “tutor01” at the beginning of this section. The model opens in a Web browser. Click “Run and Solve”.

2. Nothing happens! The solution is still $\{x = 40, y = 30\}$. This can be seen immediately from Figure 1: The `Mounting` capacity is not the limiting resource. So, extending it, does not change anything. To verify this, change 480 to 500 in the model and run it. Then look at the values of the variables again.

3. Change the constraint `Order2` to `y >= 45`. Then run the model again. The solution indicates that 45 units of *Jules* are produced, but the number of *Marie* drops to 25, and the overall revenue also drops to 16500. This can be seen by clicking the blue node `revenue` while 'TABLE' is the active tab.

4. The storage requirement can be added to the model by defining an additional constraint, which is

   ```
       constraint Storage:    7*x + 2*y  <=  300;
   ```

   The optimal solution is: 32 of Marie and 38 of Jules. Hence, the storage capacity limits the production substantially.

2 Names and Comments (tutor02)

—- Run LPL Code , HTML Document –

Problem: This is exactly the same model as `tutor01.lpl`. Comments are added and some simple expressions are used.

Listing 2: The Complete Model implemented in LPL [2]

```
model TUTOR02 "Names and Comments";
     /* The variables are declared */
  variable Marie,x "Number of robots Marie";
    Jules,y        "Number of robots Jules";
    /*  The constraints are listed */
  constraint
    Component: 5*x + 5*y <= 300+50   "Capacity of comp per week";
    Mounting:  4*x + 8*y <= 500-20   "Capacity of mount per week";
    Testing:   6*x + 2*y <= 30*10    "Capacity of test per week";
    Order1:      x       >= 200/10   "Already ordered of Maries";
    Order2:            y >= 4^1.9534 "Already ordered of Jules'";
    // This function is maximized
  maximize revenue: 300*x + 200*y;
    -- Finally, three data tables are written to the NOM-file
  Writep(revenue, x, y);
end
```

LPL Modeling Steps

One can add comments, blanks or linefeeds everywhere between tokens (words). Furthermore,

1. An entity can have more than one name: Variable *Marie*, e.g., has two names: `Marie` and `x`. This will be especially useful for indexes (see later). A second name is introduced by adding it after the first name separated by a comma.

2. Multiline comments must be within / * and * /. They can be nested.

3. Short one-line comments begin with a double-dash (`--`) or with `//`. They end with an end-of-line.

4. Expressions, like $300 + 50$, can be used.

5. The 5 operators used here are: `+` (add), `-` (minus), `*` (times), `/` (divide) and `^` (power).

6. `maximize` calls a default solver library or program and reads the results back into the LPL.

7. `Write` writes the results to the so called `NOM-file` (here the filename is: `tutor02.nom`). In `lplw.exe` this file is loaded automatically after a run and shown in a new tab.

Questions

1. What happens if one adds two dashes just before the word `Testing` ?

2. Outcomment the line of `Jules,y ...`. What happens?

3. What happens if we change the name `Marie` to `Mary` ?

Answers

1. The constraint `Testing` is commented and does not taking any effect when resolving the problem. That is, the solution will be now : $\{x = 55, y = 15\}$.
2. Running the model will stop at the first occurrence of y and an error message is output at the status line: *Error: 516 Unknown identifier*.
3. Nothing! The name is not used elsewhere in the model.

3 Using Indices I (tutor03)

—- Run LPL Code , HTML Document –

Problem: [[This model now introduces the very fundamental concept of *index*.]] Suppose, we have 10 different types of robots (not just 2 like in the previous models `tutor01` and `tutor02`. A way to deal with this is to introduce 10 different variables. However there is a more economical way, that is, using indexes. In mathematical notation, one can write this as follows:

$$x_i, \quad \text{with } i \in I = \{1 \ldots 10\}$$

In LPL syntax, a very similar notation is used:

```
    set I := 1..10;
    variable x{i in I};
```

In the same way, in mathematics, one used the summation notation to add all variables. The "mounting robots" capacity constraint can be formulated as:

$$\sum_{i \in I} HM_i \cdot x_i \leq 4800$$

where HM_i is the time spent to mount a single robot type i, 4800 being the total capacity for this production step. In LPL, again, this is formulated close to this notation as follows (note that every constraint has a name in LPL, here `Mounting`, to identify it within in the LPL code):

```
  Mounting:  sum{i in I} HM[i] * x[i] <= 4800;
```

Listing 3: The Complete Model implemented in LPL [2]

```
model TUTOR03 "Using Indices I";
  set  I := 1..10  "A set with 10 elements";
  integer variable  x{i in I} "The number of different type of robots";
  parameter HC{i in I} := [ 5 5 4 5 6 5 7 8 4 7 ] "Component time";
     HM{i in I} := [ 4 8 5 6 4 8 7 6 5 3 ] "Mounting time";
     HT{i in I} := [ 6 2 4 6 3 4 5 2 5 3 ] "Testing time";
     Ordered{i in I} := [ 20 15 7 6 5 8 9 8 7 5 ] "Quantity ordered";
     Price{i in I}  := [300 200 100 50 50 100 200 100 400 200 ];
  constraint
    Component: sum{i in I} HC[i] * x[i] <= 3500 "Component building";
    Mounting:  sum{i in I} HM[i] * x[i] <= 4800 "Mounting robots";
    Testing:   sum{i in I} HT[i] * x[i] <= 3000 "Testing robots";
    Order{i in I}:  x[i]                >= Ordered[i] "Ordered";
  maximize revenue: sum{i in I} Price[i] * x[i] "Maximize the revenue";
  Writep(revenue, x, HC, HM, HT);
end
```

LPL Modeling Steps

The new elements in this model are `set` and the index-operator `sum`.

1. The `set` entity declares an *index-set* called I. The index-set I has 10 elements.

$$I = \{1, \ldots, n\} \quad , \quad (n = 10)$$

2. `variable` introduces a variable *list* called `x`, which is indexed over $i \in I$. `i` is called an index. This declares the 10 variables: `x[1]` ... `x[10]`.

3. `parameter` introduces five data lists, all indexed over I. (For example, `HC[3]` (which is 4, the third data in the list) says how many hours it take to manufacture the components for the third robot type 3.)

4. The data list is directly assigned to the parameters. Hence, we have `Price[1]=300` and `Price[6]=100`, for example. These data are data vectors.

5. Indexed items can be summed up with the `sum` operator. Hence,

   ```
   HC[1]*x[1] + ... + HC[10]*x[10]
   ```

   is written as: `sum{i in I} HC[i]*x[i]`. This is LPL's notation for:

   $$\sum_{i\in\{1\ldots 10\}} HC_i \cdot x_i$$

6. Much like the summation through the `sum` operator, whole constraint-classes can be written as indexed constraints. For example:

   ```
   Order{i in I}: x[i] >=  Ordered[i];
   ```

   declares 10 constraints:

   ```
   x[1] >= Ordered[1],
   x[2] >= Ordered[2],
   ...
   ```

   In mathematical notation, we would write this as follows:

   $$x_i \geq Ordered_i \ , \quad \forall i \in I = \{1 \ldots 10\}$$

7. A `Writep` instruction can be used to write the results to the `NOM-file` which is the result file by default. In this case, resulting tables are written to the file `tutor03.nom`.

Questions

1. Replace `1..10` by `1..11` in the set definition of `I`. What happens?

2. Change `Price{i in I}` to `Price{i in J}` in the price declaration. What happens?

3. Which step of the production (production of the components, mounting or testing) is limiting the production? Is there a constraint that could be removed without changing the final result?

4. The company optimized its testing procedure and can reduce the testing time by 10% for each of the robot type. How does this affect the final result and why?

5. A market study shows that it is very unlikely to sell more than 50, 30, 40, 20, 15, 10, 60, 20, 17 and 10 units of the 10 robot types. Add the corresponding constraint and explain why the result changes dramatically.

6. Instead of maximizing the total revenue only, we take the material costs into account too. The material costs per type and unit are 120, 120, 20, 15, 9, 30, 10, 27, 270 and 20 respectively. Change the model in order to maximize the revenue.

7. Which type of robot generates the largest revenue? Use an indexed operator to calculate the right answer directly in LPL.

8. Assume that a fourth production step is needed. What do you need to change in the model and where do you see difficulties?

Answers

1. Running the model, the result is the same a s before. To really get 11 products one also needs to exdend the data tables.

2. An error occurs, because `J` is not declared. Note, LPL is case-sensitive, hence, `j` and `J` are two different names.

3. Add the three parameters after the minimizing as follows and a `Writep` statement:

```
    parameter C:=sum{i in I} HC[i] * x[i];
    parameter M:=sum{i in I} HM[i] * x[i];
    parameter T:=sum{i in I} HT[i] * x[i];
    Writep(C,M,T);
```

   This calculates the used capacities. One can derive from these numbers that the `Component` and `Testing` process are at there maximal capacity limit, whereas the `Mounting` process has a small quantity of capacity left. That means: if we increase the capacity of `Mounting`, we can not produce more, since the limiting factors are `Component` and `Testing`, not the `Mounting`.

4. The third constraint can easily be replaced by:

```
    Testing: sum{i in I} HT[i]*0.9*x[i] <= 3000;
```

   Running the model and checking the the parameters of the previous question shows, that the testing capacity is no longer a limiting factor. This allows the company to increase the revenue significantly.

5. We introduce a new parameter and add a new constraint as follows:

```
  parameter Demand{i in I} := [50 30 40 20 15 10 60 20 17 10];
  constraint Dem{i in I}: x[i] <= Demand[i];
```

   It shows that the best strategy is to produce exactly the demanded amount. All the other constraints are not limiting the production anymore and the final revenue is much lower than before.

6. A new parameter is introduced and the objective function has to be adapted:

```
  parameter Cost{i in I} := [120 120 20 15 9 30 10 27 270 20];
  maximize revenue: sum{i in I} (Price[i]-Cost[i])*x[i];
```

7. The indexed operator used to answer this question is: `argmax{i in I} A[i]` where `A` is a new parameter defined as: `parameter A{i} := Price*x;` The `argmax` operator returns `i` if the value of the `i-th` element is the largest value in the list. We find that the 9th robot generates the largest revenue.

8. We have to add a new parameter list containing the durations of the fourth step for each model. Furthermore we have to add an additional constraint. With only four steps, this is not a big deal. However, when having a lot of production steps, there exists a much better solution, that will be presented in model tutor05[1].

[1]https://lpl.matmod.ch/lpl/Solver.jsp?name=/tutor05

4 Using Indices II (tutor04)

—- Run LPL Code , HTML Document –

Problem: This is the same model as tutor03[2] with some minor syntax differences (simplifications).

Listing 4: The Complete Model implemented in LPL [2]

```
model TUTOR04 "Using Indices II";
  set  i :=[Robot1 Robot2 Robot3 Robot4 Robot5 Robot6
            Robot7 Robot8 Robot9 Robot10];
  parameter HC{i}:=[5 5 4 5 6 5 7 8 4 7];
     HM{i} := [4 8 5 6 4 8 7 6 5 3];
     HT{i} := [6 2 4 6 3 4 5 2 5 3];
     Ordered{i} := [20 15 7 6 5 8 9 8 7 5];
     Price{i} := [300 200 100 50 50 100 200 100 400 200];
  integer variable  Robots{i};
  constraint
    Component: sum{i} HC*Robots <= 3500;
    Mounting:  sum{i} HM*Robots <= 4800;
    Testing:   sum{i} HT*Robots <= 3000;
    Order{i}:  Robots   >=  Ordered;
  maximize revenue: sum{i} Price*Robots;
  Writep(revenue);
  Write('␣␣Robots␣␣␣Rob-Ord␣␣␣Price␣␣␣Robots␣␣Tot.Hours\n');
  Write{i}('␣␣%-7s␣␣␣␣%3d␣␣␣␣␣␣%4d␣␣␣␣␣␣%5d␣␣␣␣␣%2d\n',
               i,Robots-Ordered,Price,Robots,HC+HM+HT);
end
```

LPL Modeling Steps

There are several differences from the previous model `tutor03.lpl`:

1. The elements of set `i` do not need to be integers. The user can give them names such as `Robot1` ... `Robot10`.

2. If the context allows it, there is no need to make a difference between index names and set names. In this model, we use `i` for a set name *and* an index name. the syntax `i in I` can be simplified just to `i`.

3. The indices in expressions, such as `Robots[i]` can be dropped, since LPL already knows that `Robots`, for example, has been defined over the index-set `i`. Hence the two following lines are equivalent for LPL:

   ```
   sum{i} HC * Robots <= 3500
   sum{i} HC[i] * Robots[i] <= 3500
   ```

4. A `Write` statement may be used to write formatted output. In the model, we use two `Write`. The first is to output just a string to the NOM-file. The second is to output a formatted line for each `i` (explained in a later model). The syntax is similar to C or Java's `printf()` function. So, `%-7s` means to fill the first parameter (`i`) with 7 chars (left

[2]https://lpl.matmod.ch/lpl/Solver.jsp?name=/tutor03

aligned), `%3d` means to fill in an integer (the expression `Robots-Ordered`) with 3 positions, and so on.

Questions

1. Modify `%3d` in the `Write` just below `Robots` to `%6.2f` and run the model again. What's the difference?
2. How much is the revenue, if no robot has been ordered in advance?

Answers

1. The number of Robots are written with 2 decimal places.
2. 252000. One just need to outcomment the bounds `Order{i}`. The data for `Ordered` are still in the model but are no used except for the output.

5 Using Indices III (tutor05)

—- Run LPL Code , HTML Document –

Problem: A second index-set j for a set of production steps (processes) is introduced. In contrast to the previous model tutor04.lpl, where the production of the robots passed three processing steps (Component, Mounting, Testing), in this model it is supposed that the production goes through 8 steps. Instead of introducing 8 constraints, a second index-set j is introduced to collect the list of processing steps. Two data change now: a vector Capacity{j} is introduced in place of the right hand side of the constraint, and the three vectors HC{i}, HM{i}, and HT{i} in the previous model are now replaced by a two-dimensional matrix Hours{i,j}. This matrix gives the needed hours for passing the processing step j for robot type i.

Listing 5: The Complete Model implemented in LPL [2]

```
model TUTOR05 "Using Indices III";
  set  i :=[Robot1 Robot2 Robot3 Robot4 Robot5 Robot6 Robot7
            Robot8 Robot9 Robot10];
  parameter Ordered{i}:=[20 15 7 6 5 8 9 8 7 5];
     Price{i}:=[300 200 100 50 50 100 200 100 400 200];
  set j:=[Step1 Step2 Step3 Step4 Step5 Step6 Step7 Step8];
  parameter Capacity{j}:=[3500 4800 3000 3400 3000 3200 4000 2500 ];
     Hours{i,j} := [
          9      9      9      9      9      9      .      9
          5      8      2      .      3      .      .      3
          4      5      4      .      5      6      .      5
          5      6      6      .      8      .      .      4
         10      4      3      .      .      5      .      6
          5      8      4      .      .      .      5      1
          7      7      5      5      .      3      .      2
          8      6      2      4      .      .      .      5
          4      5      5      6      4      .      .      .
          7      3      3      1      1      1      1      4];
  variable  Robots{i};
  constraint
    Steps{j}: sum{i} Hours[i,j] * Robots[i] <= Capacity[j];
    Order{i}: Robots[i]  >=  Ordered[i];
  maximize   revenue: sum{i} Price[i] * Robots[i];
  Writep(revenue, Robots);
end
```

LPL Modeling Steps

The data are now organized differently: All of them are collected in "tables".

1. A second index-set (j) is introduced for the production steps. The previous models use three explicit production steps, resulting in three constraints. In this model, a generic number of steps – concretely 8 – is used.

2. The time, which indicates the hours required for each type of robot i at each production step j, is defined as a two-dimensional table Hours{i,j}. Undefined (or zero) entries are entered as a dot.

3. The Hours{i,j} matrix is defined as a list of $|i| \cdot |j|$ entries.

4. Note that the `Steps{i}` constraint generates 8 different constraints. One can verify this when generating the `EQU-file` (in lplw.exe choose Tools/Options enter 'e', run, then open menu 'Tools/EQU-file').

Questions

1. Choose Tools/Options, at the tab "Compiler" enter 'e', Run the model, then click the menu Tools/Create EQU-file. A listing of all equations opens.

2. For which constraints the capacity is used at 100%?

3. What big advantage do you see in using one data table instead of using a separate list for every constraint?

4. The production manager wants to produce the same quantity of every type of robot. What do you have to change?

Answers

1. An equation listing is generated. It is stored in the `EQU-file`. On disk the file is: `tutor06.equ`.

2. (1) Run the model, (2) click the tab 'TABLE', (3) click the blue small cycle C called 'Steps' at the left on the "Tree" tab part of the application, (4) open menu Tools/Options, (5) choose tab "Format", (6)in the "Data Attribute" list choose the radio button "Dual / Reduced cost". A reduced table appears with processing steps `Step1` and `Step3`. These two constraints are at their maximal capacities. (To understand this you need to know what dual values are.)

3. When you have to add an additional production step or an additional type of robot the above formulation is adapted much faster. Instead of adding a parameter list and an additional constraint you just have to adjust the corresponding set formulation and enter the data into the table. The additional constraint is generated automatically by LPL. With regard to reusability and maintainability, you should always prefer the structure demonstrated in this example.

4. We add a variable `quantity` and the following constraint:

   ```
   constraint Quantity{i}: Robots[i] = quantity;
   ```

   The maximum revenue now is 92968.75. Note that in this example both the variable and the constraint have the same name. However, this is only possible because LPL makes a difference between lower- and uppercase letters.

6 Data Include Files (tutor06)

—- Run LPL Code , HTML Document –

Problem: This is the same model as `tutor05.lpl`. The data are stored in another file (`tutor.inc`) and are included at parse-time. The instruction to physically read and include another file into the code is:

```
/ *$I 'filename' * /
```

The included data file – also specified in LPL syntax – is as follows (note that in the Web browser the incluse file already directly included in the code):

```
-- tutor.inc: data file for the tutor models as include-file

set  i := [Robot1 Robot2 Robot3 Robot4 Robot5 Robot6 Robot7
           Robot8 Robot9 Robot10];
     j := [Step1 Step2 Step3 Step4 Step5 Step6 Step7 Step8];
parameter Ordered{i}:=[20 15 7 6 5 8 9 8 7 5];
     Price{i} := [300 200 100 50 50 100 200 100 400 200];
     Capacity{j} := [3500 4800 3000 3400 3000 3200 4000 2500 ];
     Hours{i,j} := [
          9       9       9       9       9       9       .       9
          5       8       2       .       3       .       .       3
          4       5       4       .       5       6       .       5
          5       6       6       .       8       .       .       4
         10       4       3       .       .       5       .       6
          5       8       4       .       .       .       5       1
          7       7       5       5       .       3       .       2
          8       6       2       4       .       .       .       5
          4       5       5       6       4       .       .       .
          7       3       3       1       1       1       1       4];
```

Listing 6: The Complete Model implemented in LPL [2]

```
model TUTOR06 "Data Include Files";
  /*$I 'tutor.inc' */ -- a data file is included here
  variable  Robots{i};
  constraint
    Steps{j}: sum{i} Hours*Robots <= Capacity;
    Order{i}: Robots  >=  Ordered;
  maximize revenue: sum{i} Price*Robots;
  Write('Revenue:␣%7.3f\n',revenue);
  Write('Variable␣Lower␣Bounds:\n%10.2f\n', {i} GetValue(Robots,1));
  Write('Variable␣Upper␣Bounds:\n%10.2f\n', {i} GetValue(Robots,2));
  Write('Dual␣Values:\n%10.2f\n', {i} GetValue(Robots,3));
  Write('Element␣Names␣of␣set␣i:\n%10s\n', {i}i);
  Write('\nVariableNames␣␣␣␣Values\n-------------------------\n');
  Write{i}('%-15s␣=␣%6.2f\n',  GetName(Robots,0), GetValue(Robots,0));
end
```

LPL Modeling Steps

1. At any point, a file can be included using the `$I` option. Inclusion of up to level of five is possible.

2. The second, third and fourth `Write` statement prints the lower bound, the upper bound, and the dual values of the `Robots` variable. The function in LPL is defined as `GetValue(reference`
   (see manual).

3. The last `Write` statement prints the variable names with the values: The function `GetName()` returns the variable names as string.

Questions

1. List the right hand side of constraint `Steps`. List also the reduced costs of `Steps`.

2. Click the 'Files' tab in lplw in the left part then click `tutor.inc`.

Answers

1. You need to add the instruction

```
Write('RHS␣Values:\n%10.2f\n' , {j} (GetValue(Steps,2));
Write('RC␣␣Values:\n%10.2f\n' , {j} (GetValue(Steps,7));
```

2. The file `tutor.inc` opens in a new tabbed window and the content can be edited.

7 Reading text files (tutor07)

—- Run LPL Code , HTML Document –

Problem: The data are read from a text-file `tutor.txt` using the function `Read`.

Listing 7: The Complete Model implemented in LPL [2]

```
model TUTOR07 "Reading text files";
  set   i; j;
  parameter  Ordered{i}; Price{i}; Capacity{j};
  parameter  Hours{i,j};
  variable   Robots{i};
  constraint
    Steps{j}: sum{i} Hours*Robots <= Capacity;
    OrderX{i}:   Robots  >=  Ordered;
  parameter cnt;
  Read('tutor.txt,␣%1:Table');
  Read{i}('%1', i=1,Price,Ordered);
  Read{j}('%2', j,Capacity);
  Read{i}('%3', i, {j} Hours, cnt:=cnt+1);
  maximize revenue: sum{i} Price*Robots;
  Writep(revenue, Robots,cnt);
end
```

The text data file read by LPL has the following structure:

```
// tutor.txt: data file as plain text for the tutor models
// Data for the tutor examples

Table 1: Robots: i, Price, Ordered
   Robot1       300  20
   Robot2       200  15
   Robot3       100   7
   Robot4        50   6
   Robot5        50   5
   Robot6       100   8
   Robot7       200   9
   Robot8       100   8
   Robot9       400   7
   Robot10      200   5

Table 2: Steps: j, and Capacity
   Step1  3500
   Step2  4800
   Step3  3000
   Step4  3400
   Step5  3000
   Step6  3200
   Step7  4000
   Step8  2500

Table 3  : Hours
```

```
/*        Step1  Step2  Step3  Step4  Step5  Step6  Step7 Step8 */
   Robot1   9      9      9      9      9      9      .      9
   Robot2   5      8      2      .      3      .      .      3
   Robot3   4      5      4      .      5      6      .      5
   Robot4   5      6      6      .      8      .      .      4
   Robot5  10      4      3      .      .      5      .      6
   Robot6   5      8      4      .      .      .      5      1
   Robot7   7      7      5      5      .      3      .      2
   Robot8   8      6      2      4      .      .      .      5
   Robot9   4      5      5      6      4      .      .      .
   Robot10  7      3      3      1      1      1      1      4
```

LPL Modeling Steps

The data are separated from the model file and are organized in a text file `tutor.txt`. This file is organized sequentially in three blocks – also called “Tables” each beginning with the user defined *block delimiter string* `Table`. Each `Read` instruction can read a block. Which block to read is specified by a *block number* (`'%1'`). These blocks are numbered beginning with 1. Each `Read` opens the file, reads a block and closes the file automatically. There is no need to open and close files in LPL.

1. LPL can read these blocks separately with the `Read` function. The data within the blocks must be organized in rows and columns. Such files are easy to generate by other applications.

2. The first `Read` instruction only defines two file parameters: the filename (`tutor.txt`) and the block delimiter string (`Table`). These parameters are stored for subsequent `Read` instructions.

3. The second `Read` instruction opens the file, then jumps to the first (`'%1'`) occurrence of `Table` that starts a new line and begins reading from the next line. Since `Read` is indexed over `{i}`, it reads lines until another block delimiter string appears at the beginning of a line or until the end of file has been reached. Each line contains three data separated by blanks. They are recognized and assigned to the internal entities `i`, `Ordered`, and `Price`.

4. A single `Read` always reads a single line. An indexed `Read` repeats reading lines from a block. Hence, `Read{i}` repeats the reading. The read elements are separated by commas.

5. If an element to read is indexed then the elements are repeatedly read on the same line. Hence, the code `i, {j}Hours` reads first the `i` on the line and then repeats reading the `Hours[i,j]` on the i-th line ordered by `j`. It is a powerful mechanism to read entire matrices in a single instruction.

6. Furthermore, the last instruction displays an instruction `cnt:=cnt+1` that is executed after each reading an line from the file.

Questions

1. Exchange the second and the third `Read` instruction. What happens?
2. Comment the `maximize` instruction then execute. What happens?

Answers

1. Nothing! The reads can be done in any order. There is no need to read the text blocks in sequential order from a text file.
2. The model is not solved. However, that data are nevertheless read from the textfile. This can be seen when adding an instruction `Writep(Hours)`, for example. Another way to see the data is to click the yellow cycle 'M' above 'Tree' in the left part of the lplw window. Clicking on it enlarges the list by green diamond signs (for example). Click on 'Hours' then on the 'TABLE' tab in the menu and the data of `Hours` are displayed in a grid.

8 Reading data from Excel I (tutor07a)

—- Run LPL Code , HTML Document –

Problem: The data are read from an Excel sheet (Sheet2) by the instruction `Read`. Note that lpl should correspond to the Excel Version 32/64-bit to run correctly.

Listing 8: The Complete Model implemented in LPL [2]

```
model TUTOR07a "Reading data from Excel I";
  set   i; j;
  parameter  Ordered{i}; Price{i}; Capacity{j};
  parameter  Hours{i,j};
  variable   Robots{i};
  constraint Steps{j}: sum{i} Hours*Robots <= Capacity;
    OrderX{i}: Robots  >=  Ordered;
  string parameter WB:='rdb:tutor.xlsx';
  Read{i}(WB&',[Sheet2$A5:C15]',
                  i='Robots', Price='Price', Ordered='Ordered');
  Read{j}(WB&',[Sheet2$E5:F13]', j='Steps', Capacity='Capacity');
  Read{i,j}(WB&',[Sheet2$H5:J62]', i='i', j='j', Hours='Hours');
  maximize revenue: sum{i} Price*Robots;
  Writep(revenue, Robots);
end
```

Note that the data in the Excel sheet `Sheet2` are organized in a similar way than the data in a database (in contrast to the sheet `Sheet1`). The Excel sheet `Sheet2` contains three “tables” ordered in *ranges* in Excel with a header in each range. Therefore, the data can be read from Excel in the same way as they are read from a regular relational database using a database driver. The whole information of which driver to use is packed by the filename define in `WB`

```
  string parameter WB:='rdb:tutor.xls';
```

The starting part `rdb:` says to use a database driver and the file extension says to use the Microsoft Jet driver. The parameter is expanded by LPL to the following connection string (and one could alterantively use this connection string):

```
string parameter WB:=Provider=Microsoft.Jet.OLEDB.4.0;Extended\
  Properties="Excel 8.0;HDR=Yes;";Data Source=tutor.xls';
```

(Note that the \ character at the end of the first line as well as all leading blanks are removed by the LPL parser.)

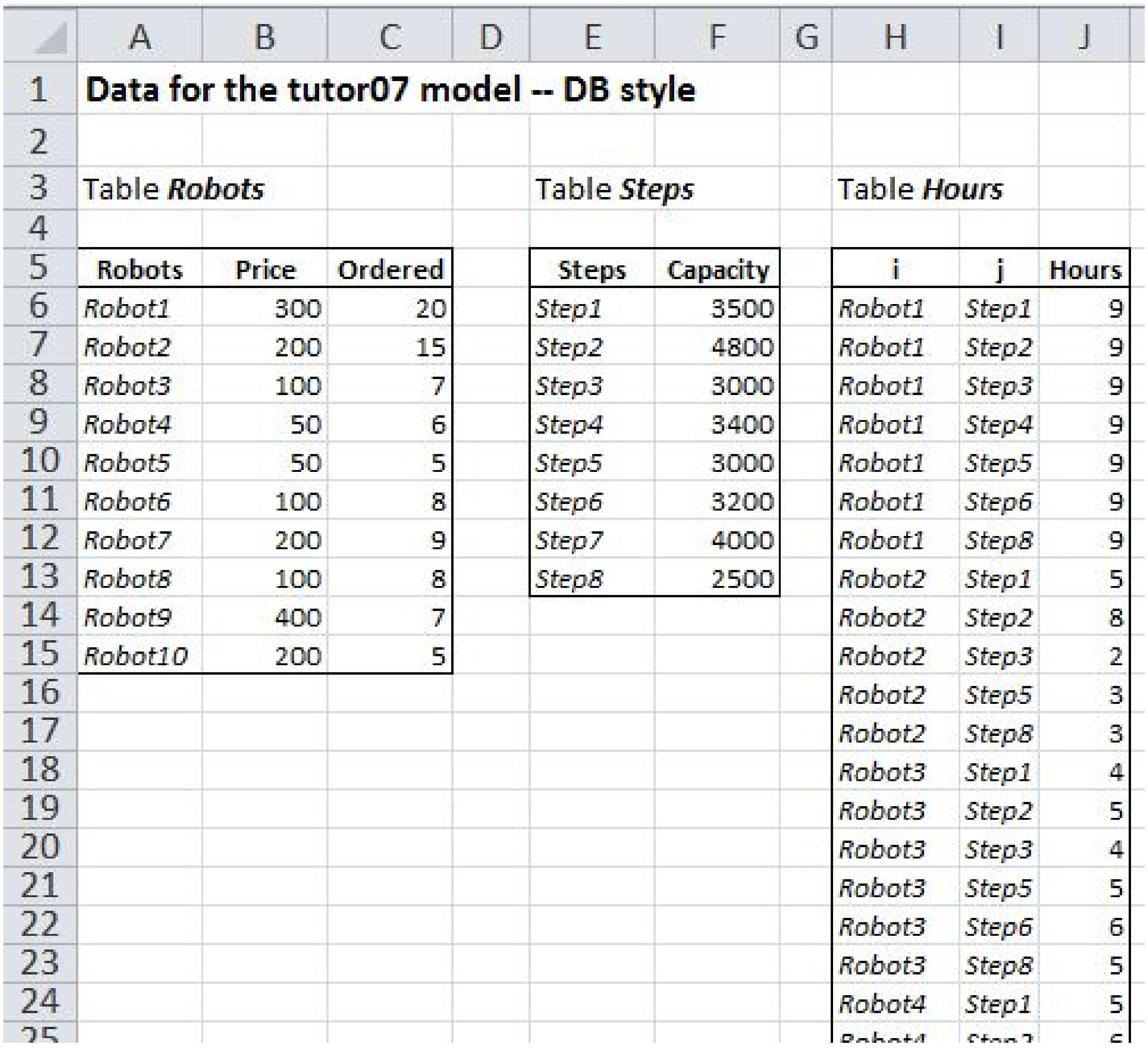
The Excel sheet is shown in Figure 2.

LPL Modeling Steps

The data are separated from the model file and can be read by instruction. The `Read` instructions are similar than they are in the previous version where data are read from text files. There are differences:

1. An string parameter `WB` defines *the connection string*[3] to Excel. This is concatenated with the source filename (here an Excel workbook) do give the string `WB`. This defines the *source* of reading. (Note that a string can be broken over lines in LPL using a \ character at the end of a line.)

[3]The concept of *connection string* is explained at wikipedia or a similar source. The user should be familiar with this concept in order to use database connectivity with LPL.



	A	B	C	D	E	F	G	H	I	J
1	**Data for the tutor07 model -- DB style**									
2										
3	Table ***Robots***				Table ***Steps***			Table ***Hours***		
4										
5	**Robots**	**Price**	**Ordered**		**Steps**	**Capacity**		**i**	**j**	**Hours**
6	*Robot1*	300	20		*Step1*	3500		*Robot1*	*Step1*	9
7	*Robot2*	200	15		*Step2*	4800		*Robot1*	*Step2*	9
8	*Robot3*	100	7		*Step3*	3000		*Robot1*	*Step3*	9
9	*Robot4*	50	6		*Step4*	3400		*Robot1*	*Step4*	9
10	*Robot5*	50	5		*Step5*	3000		*Robot1*	*Step5*	9
11	*Robot6*	100	8		*Step6*	3200		*Robot1*	*Step6*	9
12	*Robot7*	200	9		*Step7*	4000		*Robot1*	*Step8*	9
13	*Robot8*	100	8		*Step8*	2500		*Robot2*	*Step1*	5
14	*Robot9*	400	7					*Robot2*	*Step2*	8
15	*Robot10*	200	5					*Robot2*	*Step3*	2
16								*Robot2*	*Step5*	3
17								*Robot2*	*Step8*	3
18								*Robot3*	*Step1*	4
19								*Robot3*	*Step2*	5
20								*Robot3*	*Step3*	4
21								*Robot3*	*Step5*	5
22								*Robot3*	*Step6*	6
23								*Robot3*	*Step8*	5
24								*Robot4*	*Step1*	5

Figure 2: Data in Excel Sheet

2. The string `WB` is concatenated with the "table name", that is, the range within the Excel sheet. A range in Excel is specified by the worksheet name followed by a top-left and bottom-right cell name as follows: `[Sheet2$A5:C15]`. The de filename and table name are separated by a comma.

3. The three following parameters have the form `lpl_id = source_field`. Hence, the expression `i='Robots'` means that the set `i` is assigned from the field (column) with the header name `'Robots'`, this is a standard way to name columns in Excel that act as database tables.

4. It is important to notice that the read elements *must be* organized into columns or fields. In this case, the columns are named as `'Robots'`, `'Ordered'`, and `'Price'`. For LPL, they also could be accessed through numbers `1`, `2`, and `3`, indicating the column numbering beginning with number `1`. Hence, the instruction could also be written as follows:

```
    Read{i}(WB&',[Sheet2$A5:C15]', i=1, Price=2, Ordered=3);
```

indicating that `i` is read from "column" `1` in the indicated sheet range. If the data are *not* organized this way, one can read from Excel using a more direct driver (see model tutor07a1[4]).

5. Since the three elements are listed in increasing numbers beginning with 1, one also can leave them out, LPL just assign them anyway. Hence the instruction can be simplified to:

```
    Read{i}(WB&',[Sheet2$A5:C15]', i, Price, Ordered);
```

Now we are basically on the same instruction as for the text reading in the previous model tutor07[5], except that the source name is different.

Questions

1. Replace the first `Read` instruction to:

```
    Read{i}(WB&',[Sheet2$A5:C15]', i, Price, Ordered);
```

2. Replace the first `Read` instruction to:

```
    Read{i}('tutor.txt,%1:Table', i, Price, Ordered);
```

Compare the two statements and compare the source file too.

Answers

1. This verifies that the two instruction are indeed the same.

2. The same data are read from the text file and the Excel sheet. This shows: text file reading and Excel reading are – from the syntactical point of view – basically the same. Only the source parameter has changed. One also can write for text files:

```
    Read{i}('tutor.txt,%1:Table', i=1, Price=2, Ordered=3);
```

This confirms that in text files "tables" are also seen as columns and rows. In text files the columns are delimited by certain "blank" characters. By default, they are *space*, *tabulator* and *linefeed*. However the user can define its own delimiting characters. Common are *comma* or *semicolon*.

The Excel Worksheet can be downloaded here: tutor.xls.

[4]https://lpl.matmod.ch/lpl/Solver.jsp?name=/tutor07a1
[5]https://lpl.matmod.ch/lpl/Solver.jsp?name=/tutor07

9 Reading data from Excel II (tutor07a1)

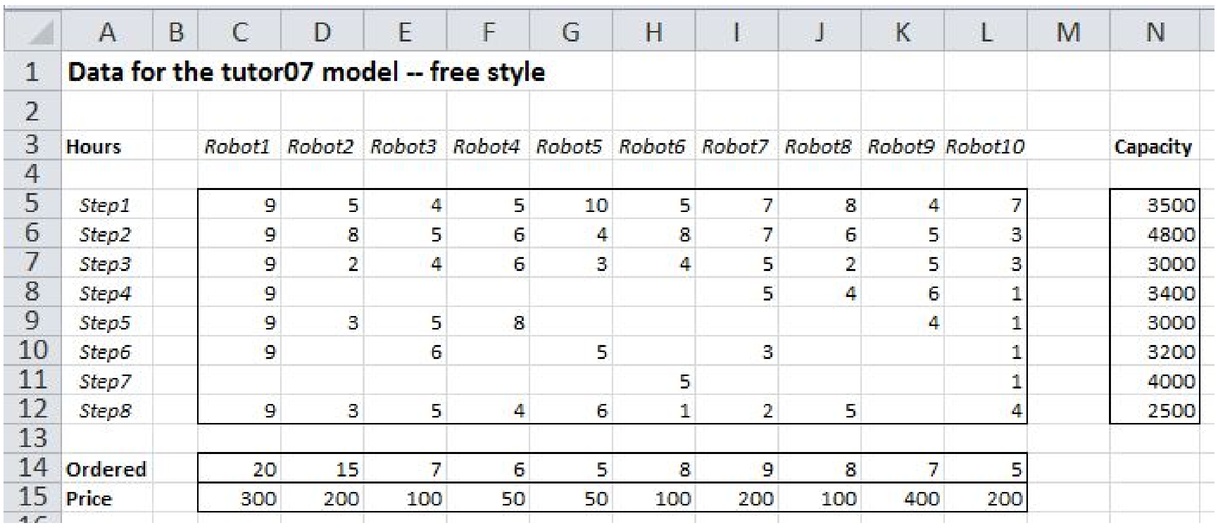
—- Run LPL Code , HTML Document –

Problem: The data are read from an Excel sheet by the instruction `Read`. Note that lpl should correspond to the Excel Version 32/64-bit to run correctly.

Listing 9: The Complete Model implemented in LPL [2]

```
model TUTOR07a1 "Reading data from Excel II";
  set   i; j;
  parameter  Ordered{i}; Price{i}; Capacity{j};
  parameter  Hours{i,j};
  variable   Robots{i};
  constraint
    Steps{j}: sum{i} Hours * Robots <= Capacity;
    OrderX{i}:   Robots  >=  Ordered;
  string parameter XLS := 'tutor.xls';
  Read(XLS&',[Sheet1$C3:L3]', {i}i);
  Read(XLS&',[Sheet1$C14:L14]', {i}Ordered);
  Read(XLS&',[Sheet1$C15:L15]', {i}Price);
  Read{j}(XLS&',[Sheet1$A5:A12]', j);
  Read{j}(XLS&',[Sheet1$N5:N12]', Capacity);
  Read{j}(XLS&',[Sheet1$C5:N12]', {i}Hours);
  maximize revenue: sum{i} Price*Robots;
  Writep(revenue, Robots);
end
```

Compared to the data in the previous Excel `tutor.xls$Sheet2` (see model tutor07a[6]), the data in this worksheet are organized in a more "free style" (Figure 3. The data are loosely scattered over the sheet.



	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	Data for the tutor07 model -- free style													
2														
3	Hours		Robot1	Robot2	Robot3	Robot4	Robot5	Robot6	Robot7	Robot8	Robot9	Robot10		Capacity
4														
5	Step1		9	5	4	5	10	5	7	8	4	7		3500
6	Step2		9	8	5	6	4	8	7	6	5	3		4800
7	Step3		9	2	4	6	3	4	5	2	5	3		3000
8	Step4		9						5	4	6	1		3400
9	Step5		9	3	5	8					4	1		3000
10	Step6		9		6		5		3			1		3200
11	Step7							5				1		4000
12	Step8		9	3	5	4	6	1	2	5		4		2500
13														
14	Ordered		20	15	7	6	5	8	9	8	7	5		
15	Price		300	200	100	50	50	100	200	100	400	200		

Figure 3: Data in Excel Sheet ("free style")

The Excelsheet `Sheet1` contains data distrubuted over several ranges. For example the list of robot types is displayed in the cells `C3:L3`. The set of production steps is in the range `A5:A12`, and so on. These ranges can be read by LPL, each with a separate `Read` statement.

The Excel Worksheet can be downloaded here: tutor.xls.

[6]https://lpl.matmod.ch/lpl/Solver.jsp?name=/tutor07a

LPL Modeling Steps

The data are separated from the model file and can be read by instruction. The `Read` instructions are similar to those in the previous version where data are read from text files. There are differences:

1. Each `Read` instruction contains two parameters: the first parameter defines the Excel Filename and the "table" separated by a comma. The "table" consists of a worksheet name and a range, enclosed within brackets. The second parameter is LPL entity that must be read.
2. Again an indexed `Read` reads over several "lines" (that is, *rows*) in a sheet, and an indexed parameter reads cells over several *columns*.
3. Hence, the parameter `{i}Ordered` means to read cells on a single row, defined by the range `C14:L14` (see second `Read`).
4. Whereas, `Read{j}('tutor.xls,[Sheet1$A5:A12]', j);` means to read a single data `j` on several rows. The logic is similar for other `Read` statements.

Questions

1. What is the main difference between the "free style" and the "database style" of data in Excel?

Answers

1. In the "free style", the data are scattered over the sheet and LPL must read each range separately. In "database style" the tables are organized in a way similar to a database table (with a header as fieldnames) and LPL reads them using a database driver, like ODBC or MS's Jet engine.

10 Reading data from Database (tutor07b)

—- Run LPL Code , HTML Document –

Problem: The data are read in from an Access database by the instruction `Read`. Note that the LPL app should correspond to the MS Access Version 32/64-bit to run correctly.

Listing 10: The Complete Model implemented in LPL [2]

```
model TUTOR07b "Reading data from Database";
  set   i; j;
  parameter  Ordered{i}; Price{i}; Capacity{j};
  parameter  Hours{i,j};
  variable   Robots{i};
  constraint
    Steps{j}: sum{i} Hours * Robots <= Capacity;
    OrderX{i}:    Robots   >=  Ordered;
  string parameter DB:='tutor.mdb';
  Read{i}(DB&',iTable', i='Robots', Price='Price', Ordered='Ordered');
  Read{j}(DB&',jTable', j='Steps', Capacity='Capacity');
  Read{i,j}(DB&',HoursTable', i='i', j='j', Hours='Hours');
  //Write{i}('&:'&DB&'test.mdb,HoursTable1', '', 'I'=i,{j} (j=Hours));
  maximize revenue: sum{i} Price*Robots;
  Writep(revenue, Robots);
end
```

The MS Access database `tutor.mdb` consists of three tables as shown in Figure 4. It can be downloaded here: tutor.mdb.

LPL Modeling Steps

The data are separated from the model file and can be read by instruction. The `Read` instructions are similar to those in the previous versions where data are read from text files and Excel sheets. There are differences:

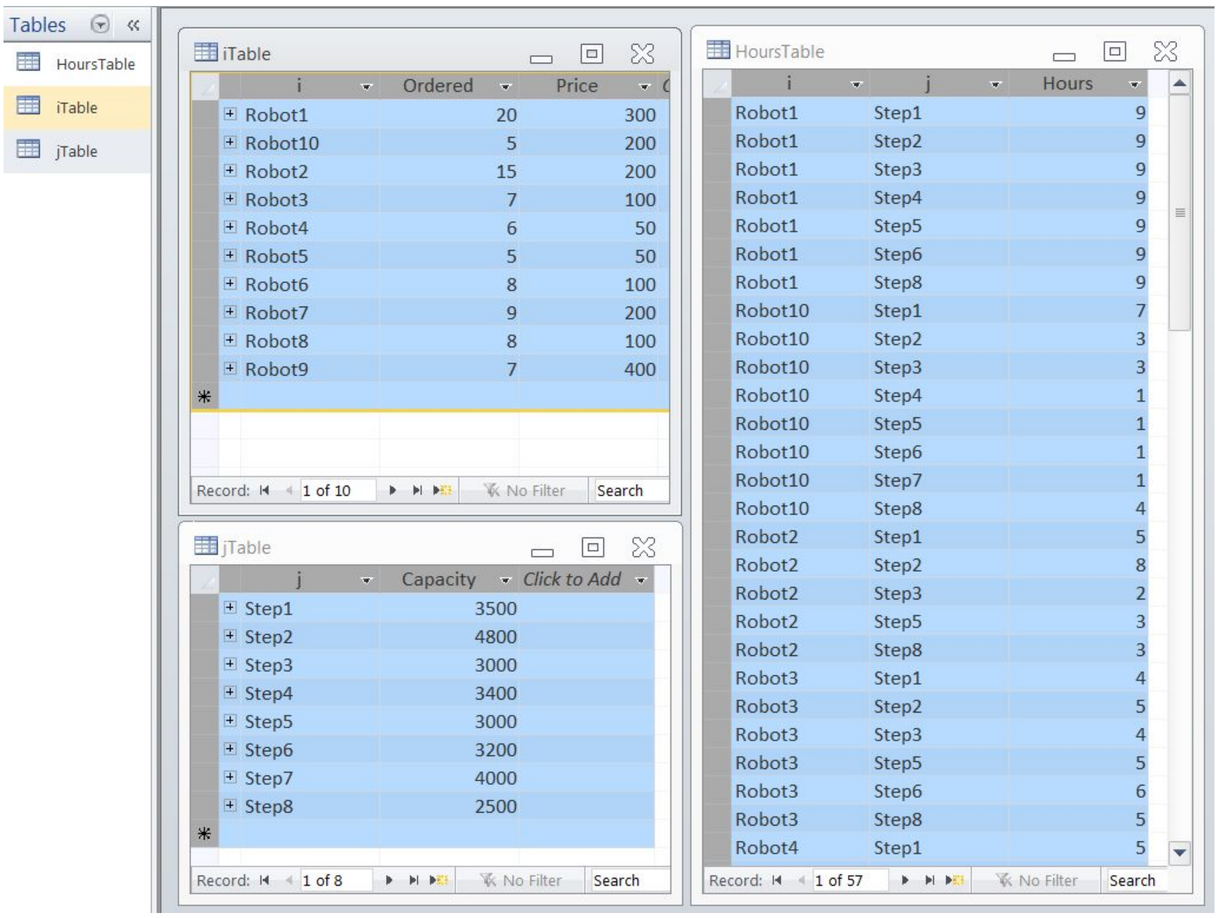
1. An string parameter `DB` defines *the connection string* to a database. [7] In LPL, the connection string is derived from the file extension. In this case it is `.mdb` that is a Access database and so the parameter is automatically expanded to:

   ```
   string parameter DB:='Provider=Microsoft.Jet.OLEDB.4.0;Data␣
       Source=tutor.mdb';
   ```

   This is concatenated with the source filename (here an database filename) do give the string `DB`. This defines the *source* of reading. (Note if the file extension would have been `.db` then LPL would read from a **SQLite** database, see reference manual for more details.

2. The string `DB` is concatenated with a database *table name* or a SQL `SELECT` statement that delivers a table. This is `iTable`, for example, for the first `Read` statement which contains the three fields `Robots`, `Price`, and `Ordered`. The database filename and table name are separated by a comma. Database filename and table name together specify the source of reading databases for LPL.

[7]The concept of *connection string* is explained at wikipedia or a similar source. The user should be familiar with this concept in order to use database connectivity with LPL.



iTable

i	Ordered	Price
Robot1	20	300
Robot10	5	200
Robot2	15	200
Robot3	7	100
Robot4	6	50
Robot5	5	50
Robot6	8	100
Robot7	9	200
Robot8	8	100
Robot9	7	400

Record: 1 of 10 No Filter Search

jTable

j	Capacity	Click to Add
Step1	3500	
Step2	4800	
Step3	3000	
Step4	3400	
Step5	3000	
Step6	3200	
Step7	4000	
Step8	2500	

Record: 1 of 8 No Filter Search

HoursTable

i	j	Hours
Robot1	Step1	9
Robot1	Step2	9
Robot1	Step3	9
Robot1	Step4	9
Robot1	Step5	9
Robot1	Step6	9
Robot1	Step8	9
Robot10	Step1	7
Robot10	Step2	3
Robot10	Step3	3
Robot10	Step4	1
Robot10	Step5	1
Robot10	Step6	1
Robot10	Step7	1
Robot10	Step8	4
Robot2	Step1	5
Robot2	Step2	8
Robot2	Step3	2
Robot2	Step5	3
Robot2	Step8	3
Robot3	Step1	4
Robot3	Step2	5
Robot3	Step3	4
Robot3	Step5	5
Robot3	Step6	6
Robot3	Step8	5
Robot4	Step1	5

Record: 1 of 57 No Filter Search

Figure 4: Data in the database

3. The three following parameters have the form `lpl_id = source_field`. Hence, the expression `i='Robots'` means that the set `i` is assigned from the field with the name `'Robots'` in the database table.

4. It is important to notice that the read elements are organized into fields with a certain ordering. In this case, the fields are named as `'Robots'`, `'Ordered'`, and `'Price'` in this order. For LPL, they also could be accessed through numbers `1`, `2`, and `3`, indicating the column numbering beginning with number `1`. Hence, the instruction could also be written as follows:

   ```
   Read{i}(DB&',iTable', i=1, Price=3, Ordered=2);
   ```

   indicating that `i` is read from column `1` - (Note that `Price` is the third fields no the second!)

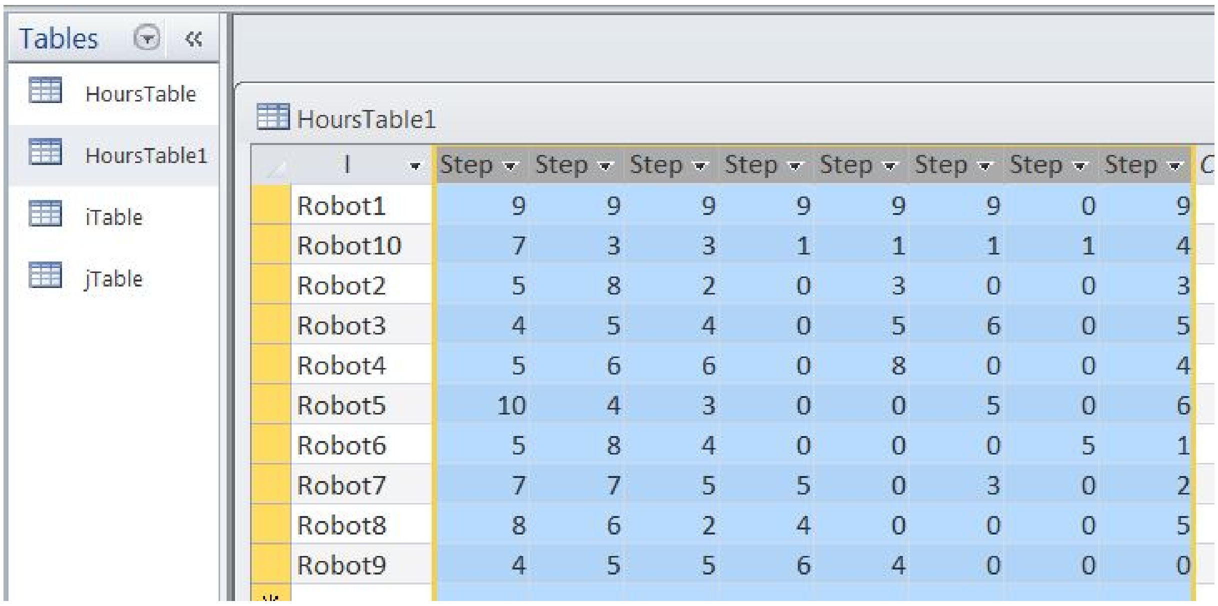
5. Reading the three elements in increasing numbers beginning with 1, one also can leave them out, LPL just assigns them anyway. Hence the instruction can be simplified to:

   ```
   Read{i}(DB&',iTable', i, Ordered, Price);
   ```

   (Note that `Ordered` is before `Price` in the database table.) Now we have basically the same instruction as for the text reading in the previous model tutor07[8]. Only the source name is different.

[8]https://lpl.matmod.ch/lpl/Solver.jsp?name=/tutor07

Further Comments: Sometimes it makes sense to read several fields into the same matrix within LPL. Suppose the database above contains an additional table which is depicted in Figure 5.



I	Step	Step	Step	Step	Step	Step	Step	Step	C
Robot1	9	9	9	9	9	9	0	9	
Robot10	7	3	3	1	1	1	1	4	
Robot2	5	8	2	0	3	0	0	3	
Robot3	4	5	4	0	5	6	0	5	
Robot4	5	6	6	0	8	0	0	4	
Robot5	10	4	3	0	0	5	0	6	
Robot6	5	8	4	0	0	0	5	1	
Robot7	7	7	5	5	0	3	0	2	
Robot8	8	6	2	4	0	0	0	5	
Robot9	4	5	5	6	4	0	0	0	

Figure 5: Table HoursTable1 in the database

This table `hoursTable1` contains the same information as the table `hoursTable`, but the layout is different. The production steps `j` are defined as fields `Step1` to `Step8`. We would like – similar to the third table in model tutor07[9] – to read all these fields into a single matrix `parameter H{i,j}` (which is identical to the table `Hours{i,j}` in our model). This can be achieved by the following reading statement

```
Read{i}(DB&',HoursTable1', i='I', {j} H);
```

Note that the syntax is similar to the syntax that is used in the model tutor07[10] for reading the `Hours` table. The condition for this to work is, of course, that the fields are in the same order as the ordering of the elements in `j`.

Questions

1. Replace the first `Read` instruction to:

```
    Read{i}(DB&',iTable', i, Ordered, Price);
```

2. Replace the first `Read` instruction to:

```
    Read{i}('tutor.txt,%1:Table', i, Price, Ordered);
```

   Compare the two statements and compare the source file too.

3. The table name can also be a SQL `SELECT` statement. Modify the LPL `READ` instruction in that sense.

Answers

1. This verifies that the two instruction are indeed the same.

[9]https://lpl.matmod.ch/lpl/Solver.jsp?name=/tutor07
[10]https://lpl.matmod.ch/lpl/Solver.jsp?name=/tutor07

2. The same data are read from the text file and the Excel sheet. This shows: text file reading and Excel reading are basically the same. Only the source parameter has changed. One also can write for text files:

   ```
   Read{i}('tutor.txt,%1:Table', i=1, Price=2, Ordered=3);
   ```

   This confirms that in text files "tables" are also seen as columns. In text files the columns are delimited by certain "blank" characters. By default, they are *space*, *tabulator* and *linefeed*. However the user can define its own delimiting characters. Common are *comma* or *semicolon*.

3. The instruction is:

   ```
   Read{i}(DB&',SELECT␣Robots,Price,Ordered␣FROM␣iTable', i,
      Price, Ordered);
   ```

   (Note that the ordering of the fields `Price` and `Ordered` are reversed.)

11 Read consecutive blocks in Text files (tutor07d)

—- Run LPL Code , HTML Document –

Problem: This file shows how LPL can read a text file as continues blocks. Normally, a single read statement opens a (text)-file reads a block of data and closes the file again. Now we want to read a block a save the file pointer before closing the file, because we want to continue reading the file in the next Read statement. This is done with the '@' char at the beginning of the file name.

The first Read instruction assigns the file name (which kept for the subsequent read statments without repeating it, a reads the 6 entries on the first line of the file. Note that the filename begins with a '@' character. That means it stores a file pointer before closing the file, such that the next read statement can continue reading from the text file.

Note that it is important to assign the three sets now, otherwise the following read instructions do not know how many lines to read.

The second Read instruction then reads the next $J = 9$ lines from the file, again keeping a pointer before closing.

The third Read instruction reads the next $K = 17$ lines assigns the first column to c_k, the second column to n_k and the the next $J = 9$ token to $A_{j,k}$.
The text data file read by LPL has the following structure:

```
92 9 17 20 1 27
1 6 1
1 5 0
1 2 0
1 2 0
1 4 0
1 5 0
1 3 0
2 3 0
1 28 0
3 11 0 0 1 0 1 0 0 0 0
3 2 1 0 1 0 0 0 0 0 0
3 8 0 0 1 0 0 0 0 0 0
3 3 0 0 1 1 0 0 0 0 0
3 1 0 1 1 0 0 0 0 0 1
3 1 1 0 1 1 0 0 0 0 0
2 12 0 0 1 0 1 0 0 0 0
2 32 0 0 1 0 0 0 0 0 0
2 6 1 0 1 0 0 0 0 0 0
1 2 0 0 1 0 1 0 0 0 0
2 7 0 0 1 1 0 0 0 0 0
2 2 0 1 1 0 1 0 0 0 1
2 1 0 0 1 0 0 0 1 0 0
0 1 0 0 1 1 1 0 1 0 0
0 1 1 0 1 0 0 0 0 0 0
2 1 1 0 1 0 1 0 0 0 0
0 1 0 0 1 0 0 0 0 0 0
```

Note when using the '@' option in a Read instruction, use it also for all other Read instruction for reading the same file. To not mix them up (except you know what you do!).

Modeling Steps

Listing 11: The Complete Model implemented in LPL [2]

```
model tutor07d "Read consecutive blocks in Text files";
  set i,h,g;
      j;
      k;
  parameter
    BA; OB; SP;
    c{k};  n{k};
    r{j};  s{j};  p{j};
    A{j,k};
  model data "input data";
    parameter I; J; K; dum;
    Read('@tutor07d.txt',I,J,K,BA,OB,SP);
    i:=1..I; j:=1..J; k:=1..K;
    Read{j}('@', r,s,p);
    Read{k}('@',c,n,{j} A);
  end
end
```

12 Writing to text files (output locally only) (tutor08)

—- Run LPL Code , HTML Document –

Problem: Data and the solution are written to a text file using the function `Write`.

Listing 12: The Complete Model implemented in LPL [2]

```
model TUTOR08 "Writing to text files (output locally only)";
  /*$I 'tutor.inc' */ -- a data file is included here
  variable   Robots{i};
  constraint
    Steps{j}: sum{i} Hours*Robots <= Capacity;
    OrderX{i}:   Robots  >=  Ordered;
  maximize revenue: sum{i} Price*Robots;
  Write('tutor08out.txt','--LPL␣generated␣file\n\nTable␣1␣--␣robot␣
      types\n');
  Write{i}('%9s\t%4d\t%2d\n', i,Price,Ordered);
  Write('\nTable␣2␣--␣processing␣steps\n');
  Write{j}('%9s\t%4d\n', j,Capacity);
  Write('\nTable␣3␣--␣unit␣time␣comsumption\n');
  Write{i}('%9s␣%4d\n', i, {j}Hours);
  Write('\nTable␣4␣--␣Solution\n');
  Write{i}('%9s␣%4d\n', i, Robots);
end
```

The text output file written by LPL has the following structure:

```
--LPL generated file

Table 1 -- robot types
   Robot1  300 20
   Robot2  200 15
   Robot3  100  7
   Robot4   50  6
   Robot5   50  5
   Robot6  100  8
   Robot7  200  9
   Robot8  100  8
   Robot9  400  7
  Robot10  200  5

Table 2 -- processing steps
    Step1 3500
    Step2 4800
    Step3 3000
    Step4 3400
    Step5 3000
    Step6 3200
    Step7 4000
    Step8 2500

Table 3 -- unit time comsumption
```

```
    Robot1     9   9   9   9   9   9   0   9
    Robot2     5   8   2   0   3   0   0   3
    Robot3     4   5   4   0   5   6   0   5
    Robot4     5   6   6   0   8   0   0   4
    Robot5    10   4   3   0   0   5   0   6
    Robot6     5   8   4   0   0   0   5   1
    Robot7     7   7   5   5   0   3   0   2
    Robot8     8   6   2   4   0   0   0   5
    Robot9     4   5   5   6   4   0   0   0
   Robot10     7   3   3   1   1   1   1   4

 Table 4 -- Solution
    Robot1   20
    Robot2  266
    Robot3    7
    Robot4    6
    Robot5    5
    Robot6    8
    Robot7    9
    Robot8    8
    Robot9  420
   Robot10    5
```

LPL Modeling Steps

The instruction `Write` can be used to write formatted text files.

1. The parameters of the function `Write` as as following: The first parameter is a filename. The second parameter is a format string, and the other parameters are the data to write. The syntax is: `Write(filename,format,datalist)`.
2. Each `Write` instruction opens the file automatically and closes it at the end of the writing.
3. If the first parameter `filename` is missing then the file is opened in an appending mode, that is, the data are added automatically at the end of the existing file which name has been defined by a previous `Write` statement.
4. If no previous `Write` has defined a filenane than the file name is the NOM-file.
5. If the first parameter `filename` is *not* missing then the file is opened in a write mode, that is, an existing file with the same name is erased before writing can begin.
6. The first parameter is missing if and only if the `Write` has only one parameter or if the first (string) parameter contains a linefeed or a % chracter.
7. The second parameter `format` specifies the format of the output. It's a string. This parameter of the first `Write`, for example, is:

   `--LPL generated file\n\nTable 1\n`

   It says to write the text `--LPL generated file` to the file, then to write two linefeed characters (`\n`), then to write `Table 1` and another linefeed.

8. The format parameter of the second `Write` is: `%9s\t%4d\t%2d\n` It says to translate `i` into a string of length 9 (`%9s`), then to write a tabulator `\t`, then to replace `%4d` with `Price` with length 4, write another tab, and finally, to write `Ordered` with length 2 (`%2d`).

Note: The model tutor08e[11] lists all format specifiers that can be used in LPL.

Questions

1. Replace the second `Write` statement with:

```
    Write{i}('%-9s\t%6.2f\t%2d\n', i,Price,Ordered);
```

2. Study all specifiers!

3. Why do all `Write` statements send the data to a single file named `tutor08out.txt` ?

Answers

1. The set `i` is written in left align mode. `Price` is written as floating point number with two decimals.

2. Open the model tutor08e.

3. Only the first `Write` specifies the filename. This filename is stored for subsequent `Write` instructions. Since the first parameter of all other `Write` instructions contain the character `\n`, they are all interpreted by LPL as the `format` parameter and data is *appended* to the file.

[11] https://lpl.matmod.ch/lpl/Solver.jsp?name=/tutor08e

13 Writing data to Excel (locally only) (tutor08a1)

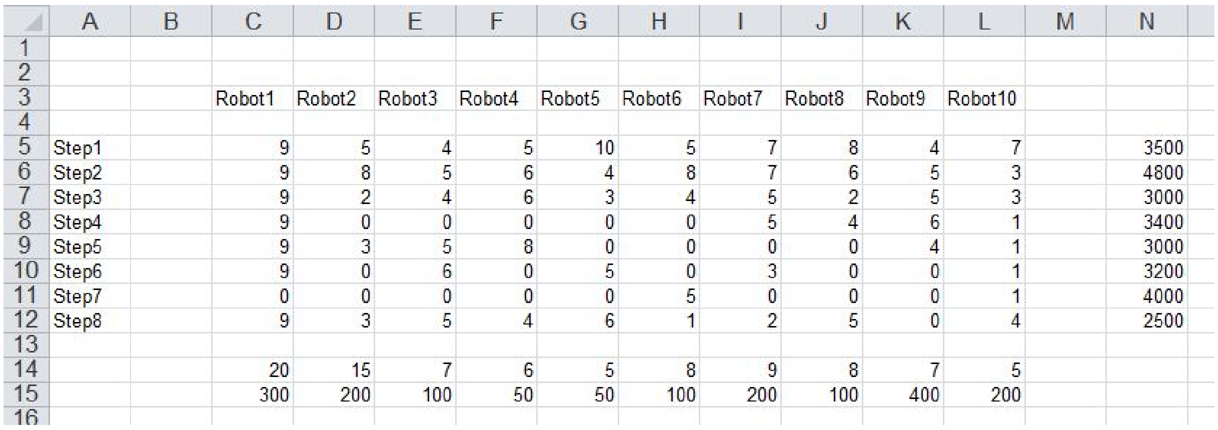
—- Run LPL Code , HTML Document –

Problem: The data are written to an Excel sheet by the instruction `Write`.

Listing 13: The Complete Model implemented in LPL [2]

```
model TUTOR08a1 "Writing data to Excel (locally only)";
  /*$I 'tutor.inc' */ -- a data file is included here
  variable   Robots{i};
  constraint
    Steps{j}: sum{i} Hours * Robots <= Capacity;
    OrderX{i}:   Robots   >=  Ordered;
  string parameter XLS := 'tutor08out.xls';
  Write(XLS&',[Sheet4$C3:L3]', {i}i);
  Write(XLS&',[Sheet4$C14:L14]', {i}Ordered);
  Write(XLS&',[Sheet4$C15:L15]', {i}Price);
  Write{j}(XLS&',[Sheet4$A5:A12]', j);
  Write{j}(XLS&',[Sheet4$N5:N12]', Capacity);
  Write{j}(XLS&',[Sheet4$C5:L12]', {i}Hours);
  maximize revenue: sum{i} Price*Robots;
  Writep(revenue, Robots);
end
```

The `Write` instructions has exactly the same syntax as the `Read` instruction when reading from Excel (see model tutor07a1[12]): only the keyword `Read` has been replaced by the keyword `Write` (and another file name – `tutor08out.xls` – is used).



	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1														
2														
3			Robot1	Robot2	Robot3	Robot4	Robot5	Robot6	Robot7	Robot8	Robot9	Robot10		
4														
5	Step1		9	5	4	5	10	5	7	8	4	7		3500
6	Step2		9	8	5	6	4	8	7	6	5	3		4800
7	Step3		9	2	4	6	3	4	5	2	5	3		3000
8	Step4		9	0	0	0	0	0	5	4	6	1		3400
9	Step5		9	3	5	8	0	0	0	0	4	1		3000
10	Step6		9	0	6	0	5	0	3	0	0	1		3200
11	Step7		0	0	0	0	0	5	0	0	0	1		4000
12	Step8		9	3	5	4	6	1	2	5	0	4		2500
13														
14			20	15	7	6	5	8	9	8	7	5		
15			300	200	100	50	50	100	200	100	400	200		
16														

Figure 6: Data in Excel Sheet ("free style")

The generated Excel sheet `Sheet4` is displayed in Figure ??. Note that if the Excel file does not exist, then a new workbook is created. Furthermore, if the corresponding worksheet (here `Sheet4`) within the Excel workbook does not exist, it is added automatically.

Questions

1. Is it possible to write data to Excel in a "database style" in LPL?

Answers

1. Unfortunately, this is not possible, because the MS Jet database driver does not correctly interpret a range such as `[Sheet1$A5:C15]` as a true database table.

[12]https://lpl.matmod.ch/lpl/Solver.jsp?name=/tutor07a1

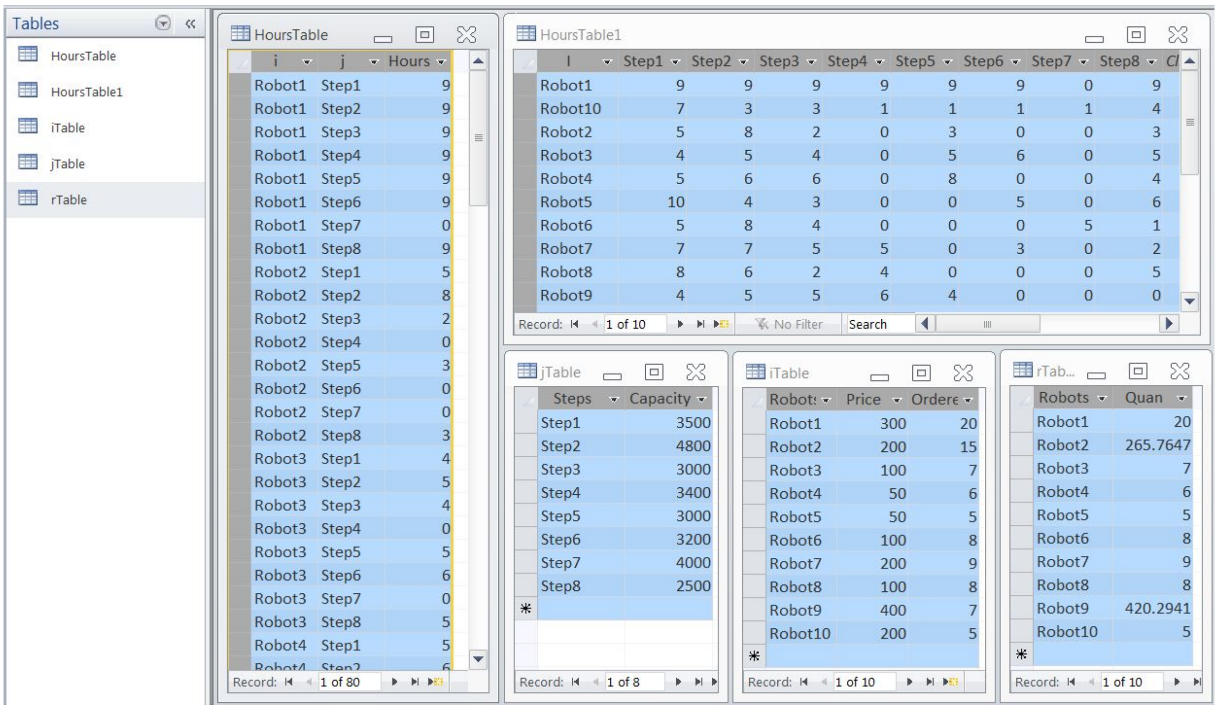
14 Writing to databases (output locally only) (tutor08b)

—- Run LPL Code , HTML Document –

Problem: Data and the solution are written to a database using the function `Write`. The `Write` statements in the model create a new database with the structure that is depicted in Figure 7.

Listing 14: The Complete Model implemented in LPL [2]

```
model TUTOR08b "Writing to databases (output locally only)";
  /* $I 'tutor.inc' */ -- a data file is included here
  variable   Robots{i};
  constraint
    Steps{j}: sum{i} Hours*Robots <= Capacity;
    OrderX{i}:   Robots  >=  Ordered;
  maximize revenue: sum{i} Price*Robots;
  string parameter  DB:='tutor08out.db';
  Write{i}   ('&:'&DB&',iTable', 'Robots'=i, 'Price'=Price,'Ordered'=
      Ordered);
  Write{i}   ('*:'&DB&',rTable', 'Robots'=i, 'Quan'=Robots);
  Write{j}   ('*:'&DB&',jTable', 'Steps'=j, 'Capacity'=Capacity);
  Write{i,j}('*:'&DB&',HoursTable', 'i'=i, 'j'=j, 'Hours'=Hours);
  Write{i}   ('*:'&DB&',HoursTable1', 'I'=i, {j} (j=Hours));
end
```


Tables

HoursTable
HoursTable1
iTable
jTable
rTable

HoursTable

i	j	Hours
Robot1	Step1	9
Robot1	Step2	9
Robot1	Step3	9
Robot1	Step4	9
Robot1	Step5	9
Robot1	Step6	9
Robot1	Step7	0
Robot1	Step8	9
Robot2	Step1	5
Robot2	Step2	8
Robot2	Step3	2
Robot2	Step4	0
Robot2	Step5	3
Robot2	Step6	0
Robot2	Step7	0
Robot2	Step8	3
Robot3	Step1	4
Robot3	Step2	5
Robot3	Step3	4
Robot3	Step4	0
Robot3	Step5	5
Robot3	Step6	6
Robot3	Step7	0
Robot3	Step8	5
Robot4	Step1	5

Record: 1 of 80

HoursTable1

I	Step1	Step2	Step3	Step4	Step5	Step6	Step7	Step8
Robot1	9	9	9	9	9	9	0	9
Robot10	7	3	3	1	1	1	1	4
Robot2	5	8	2	0	3	0	0	3
Robot3	4	5	4	0	5	6	0	5
Robot4	5	6	6	0	8	0	0	4
Robot5	10	4	3	0	0	5	0	6
Robot6	5	8	4	0	0	0	5	1
Robot7	7	7	5	5	0	3	0	2
Robot8	8	6	2	4	0	0	0	5
Robot9	4	5	5	6	4	0	0	0

Record: 1 of 10 No Filter Search

jTable

Steps	Capacity
Step1	3500
Step2	4800
Step3	3000
Step4	3400
Step5	3000
Step6	3200
Step7	4000
Step8	2500

Record: 1 of 8

iTable

Robot:	Price	Ordere
Robot1	300	20
Robot2	200	15
Robot3	100	7
Robot4	50	6
Robot5	50	5
Robot6	100	8
Robot7	200	9
Robot8	100	8
Robot9	400	7
Robot10	200	5

Record: 1 of 10

rTab...

Robots	Quan
Robot1	20
Robot2	265.7647
Robot3	7
Robot4	6
Robot5	5
Robot6	8
Robot7	9
Robot8	8
Robot9	420.2941
Robot10	5

Record: 1 of 10

Figure 7: The generated Database

In a way similar to the `Read` statement, LPL connects to a database using the *connection string* method. The parameter `DB` contains that string together with the database name `tutor08b.db`.

LPL Modeling Steps

The instruction `Write` can be used to write to existing database tables or can also create them and fill them on the fly.

1. The parameters of the function `Write` are as following: The first parameter is a connection string with the filename and a tablename separated by a comma.
2. Each `Write` instruction opens the database automatically and closes it at the end of the writing.
3. If the first parameter begins with `'&:'` then this means that the database together with the table is directly created (erasing a existing file with the same name). If it begins with `'*:'` then the existing database is extended with a new table. If it begins with `'-:'` then this means that the existing table content is deleted before new records are added. And finally, if it begins with `'+:'`, records are added to an existing content.
4. The other parameters assign the data to the specified fields: So `'Robots'=i` means to write the data of the set `i` into the fields named `'Robots'`.
5. Since the first `Write` statement is indexed over `i`, this means that LPL write as many records into the table `iTable` as `i` has elements. Each writing is an single record writing.
6. Note that the syntax of reading a record from a database using `Read` is similar (tutor07b[13]). The big difference is the inverse assignment: instead of `i='Robots'` here we write `'Robots'=i`, suggesting that the "traffic" goes from right to left as in an assignment statement.
7. Looking at the last `Write` statement, one can see that LPL also can create fields from an LPL set. The field names are directly taken from the element name of set in LPL and assigned correspondingly.

Further Comments: There are some other interesting aspects in creating databases through LPL. If the fieldname begins with the character `'I'`, for example, then LPL generates the field as a key field. If it begins with a underscore `'_'` the field will be indexed automatically in the database.

Questions

1. Combine the first two `Write` statements into a single one.
2. Add an second `Write` statement as follows:

   ```
   Write{i}('+:'&DB&',iTable', 'Robots'=i, 'Price'=Price,'Ordered'
       = Ordered);
   ```

   What happens ?
3. Replace the first `Write` statement by the following two statements:

   ```
   Write{i}('&:'&DB&',iTable', 'IRobots'=i, 'Price'=Price,'Ordered
       '= Ordered);
   Write{i}('+:'&DB&',iTable', 'IRobots'=i, 'Price'=Price,'Ordered
       '= Ordered);
   ```

   What happens ?

[13]https://lpl.matmod.ch/lpl/Solver.jsp?name=/tutor07b

Answers

1. The statement is simply:

   ```
   Write{i}('&:'&DB&',iTable', 'Robots'=i, 'Price'=Price, 'Ordered
       '= Ordered, 'Quan'=Robots);
   ```

   One only needs to add a field for the solution. There is no need to have two tables.

2. The table `iTable` is filled two times and contains 20 records, each record twice.

3. As in the previous question, the table `iTable` will be tried to fill twice. However the database integrity does not allow this since the field `'IRobots'` is now considered as a key field with unique entries only. The second `Write` will generate an database SQL exception error and the execution is stopped. The log file of LPL gives more details about the error. Look at it!

15 Creating a Report (report locally only) (tutor08c)

—- Run LPL Code , HTML Document –

Problem: This model creates a report using the Fast Report library (see FastReport).

The LPL model is almost exactly the same as the model tutor08b[14]. Data and the solution are written to a database using the function `Write`. The `Write` statements in the model create a new database with the structure that is depicted in Figure 8.

Listing 15: The Complete Model implemented in LPL [2]

```
model TUTOR08c "Creating a Report (report locally only)";
  /*$I 'tutor.inc' */ -- a data file is included here
  variable   Robots{i};
  constraint
    Steps{j}: sum{i} Hours*Robots <= Capacity;
    OrderX{i}:   Robots   >=  Ordered;
  maximize revenue: sum{i} Price*Robots;
  string parameter DB:='tutor08out.db';
  Write('&:'&DB&',tutor08c1,*' , 'a'='A␣Report', 'b'='Production', 'c'=
      'Version␣1', 'd'=Now());
  Write('*:'&DB&',hTable' , 'a'='Data␣Tables', 'b'='Production␣of␣
      Robots', 'c'='Version␣1', 'd'=Now());
  Write('*:'&DB&',i0Table', 'Robots'='ID', 'Price'='Price','Ordered'='
      Ordered', 'Quan'='Quantity');
  Write{i}('*:'&DB&',iTable', 'Robots'=i, 'Price'=Price,'Ordered'=
      Ordered, 'Quan'=Round(Robots));
  Write('*:'&DB&',j0Table', 'Steps'='ID', 'Capacity'='Capacity');
  Write{j}('*:'&DB&',jTable', 'Steps'=j, 'Capacity'=Capacity);
  Write('*:'&DB&',HoursTable0'  , 'ID'= 'ID', {j} ('a'&(j+0)=Format('%s
      ', j)));
  --Write{i,j}('*:'&DB&',HoursTable', 'i'=i, 'j'=j, 'Hours'=Hours);
  Write{i}('*:'&DB&',tutor08c2,*,*.pdf', 'I'=i,{j} (j=Hours));
end
```

In a similar way than the `Read` statement, LPL connects to a database using the *connection string* method. The parameter `DB` contains that string together with the database name `tutor08out.db`.

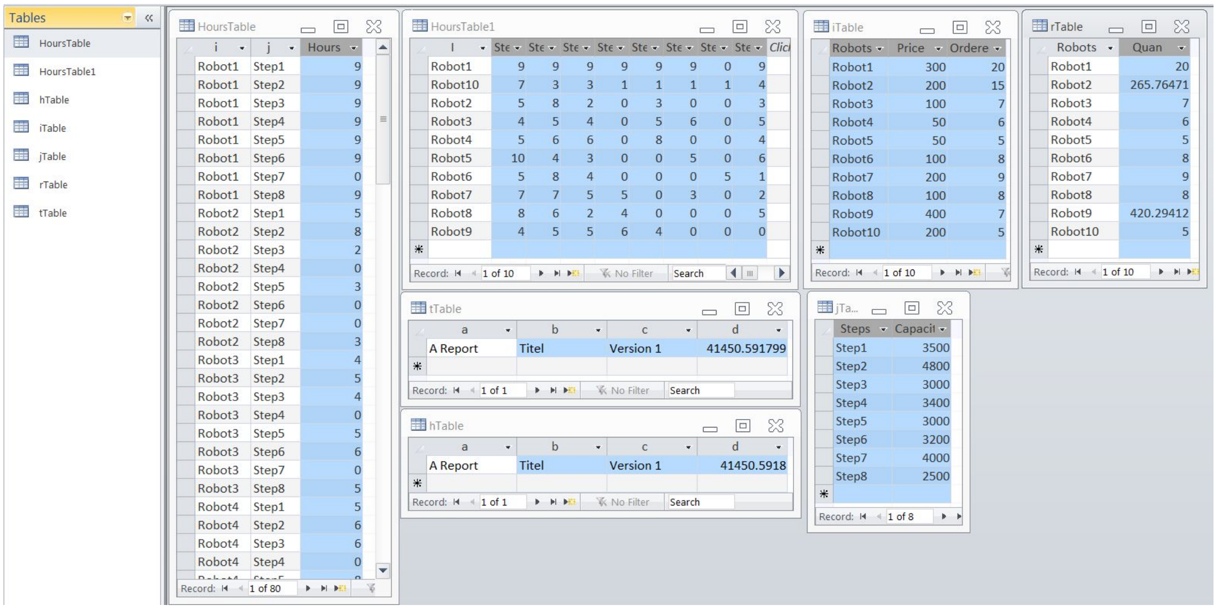
LPL Modeling Steps

In addition, a report file in PDF format is generated based on the data written to the database. The crucial modifications are:

1. The first parameter of the first `Write` statement contains additional comma followed by a star: `'&:'&DB&',tTable,*'`. LPL generates a database table `tTable`, and in addition, a *Fast Report template page* (file) called `tTable.fr3` that is editable with Fast Reports template designer. In LPLW this template page can be opened by selecting menu `View/Open Report Designer` and then selecting the file `tTable.fr3`. One can edit the page at will.

2. If the LPL model is executed a second time, then the template page is not created again, but opened as is.

[14] https://lpl.matmod.ch/lpl/Solver.jsp?name=/tutor08b



HoursTable

i	j	Hours
Robot1	Step1	9
Robot1	Step2	9
Robot1	Step3	9
Robot1	Step4	9
Robot1	Step5	9
Robot1	Step6	9
Robot1	Step7	0
Robot1	Step8	9
Robot2	Step1	5
Robot2	Step2	8
Robot2	Step3	2
Robot2	Step4	0
Robot2	Step5	3
Robot2	Step6	0
Robot2	Step7	0
Robot2	Step8	3
Robot3	Step1	4
Robot3	Step2	5
Robot3	Step3	4
Robot3	Step4	0
Robot3	Step5	5
Robot3	Step6	6
Robot3	Step7	0
Robot3	Step8	5
Robot4	Step1	5
Robot4	Step2	6
Robot4	Step3	6
Robot4	Step4	0

Record: 1 of 80

HoursTable1

I	Ste	Ste	Ste	Ste	Ste	Ste	Ste	Ste	Click
Robot1	9	9	9	9	9	9	0	9	
Robot10	7	3	3	1	1	1	1	4	
Robot2	5	8	2	0	3	0	0	3	
Robot3	4	5	4	0	5	6	0	5	
Robot4	5	6	6	0	8	0	0	4	
Robot5	10	4	3	0	0	5	0	6	
Robot6	5	8	4	0	0	0	5	1	
Robot7	7	7	5	5	0	3	0	2	
Robot8	8	6	2	4	0	0	0	5	
Robot9	4	5	5	6	4	0	0	0	

Record: 1 of 10 No Filter Search

iTable

Robots	Price	Ordere
Robot1	300	20
Robot2	200	15
Robot3	100	7
Robot4	50	6
Robot5	50	5
Robot6	100	8
Robot7	200	9
Robot8	100	8
Robot9	400	7
Robot10	200	5

Record: 1 of 10

rTable

Robots	Quan
Robot1	20
Robot2	265.76471
Robot3	7
Robot4	6
Robot5	5
Robot6	8
Robot7	9
Robot8	8
Robot9	420.29412
Robot10	5

Record: 1 of 10

tTable

a	b	c	d
A Report	Titel	Version 1	41450.591799

Record: 1 of 1 No Filter Search

hTable

a	b	c	d
A Report	Titel	Version 1	41450.5918

Record: 1 of 1 No Filter Search

jTa...

Steps	Capacit
Step1	3500
Step2	4800
Step3	3000
Step4	3400
Step5	3000
Step6	3200
Step7	4000
Step8	2500

Record: 1 of 8

Figure 8: The generated Database

3. Furthermore, the first parameter of the last `Write` statement contains also a star and a fourth substring '*.pdf' separated by a comma. It is the output file of the report. ('*.pdf' here means 'tutor08out.pdf').

```
'*:'&DB&',HoursTable1,*,*.pdf'
```

This statement creates a database table called `HoursTable1` and in addition a template page `HoursTable1.fr3`. This page also can be opened using LPLW menu entry `View/Open Report Designer`. A preview of the designer page if shown in Figure 9.

4. The four part in the first parameter (which is: `tutor08out.pdf`) is the filename of the report file.

5. If the extension of the report file were `docx` – hence, `tutor08out.docx` – then automatically a docx (MS-Word readable) format file would be generated. Other format are defined, see manual.

Questions

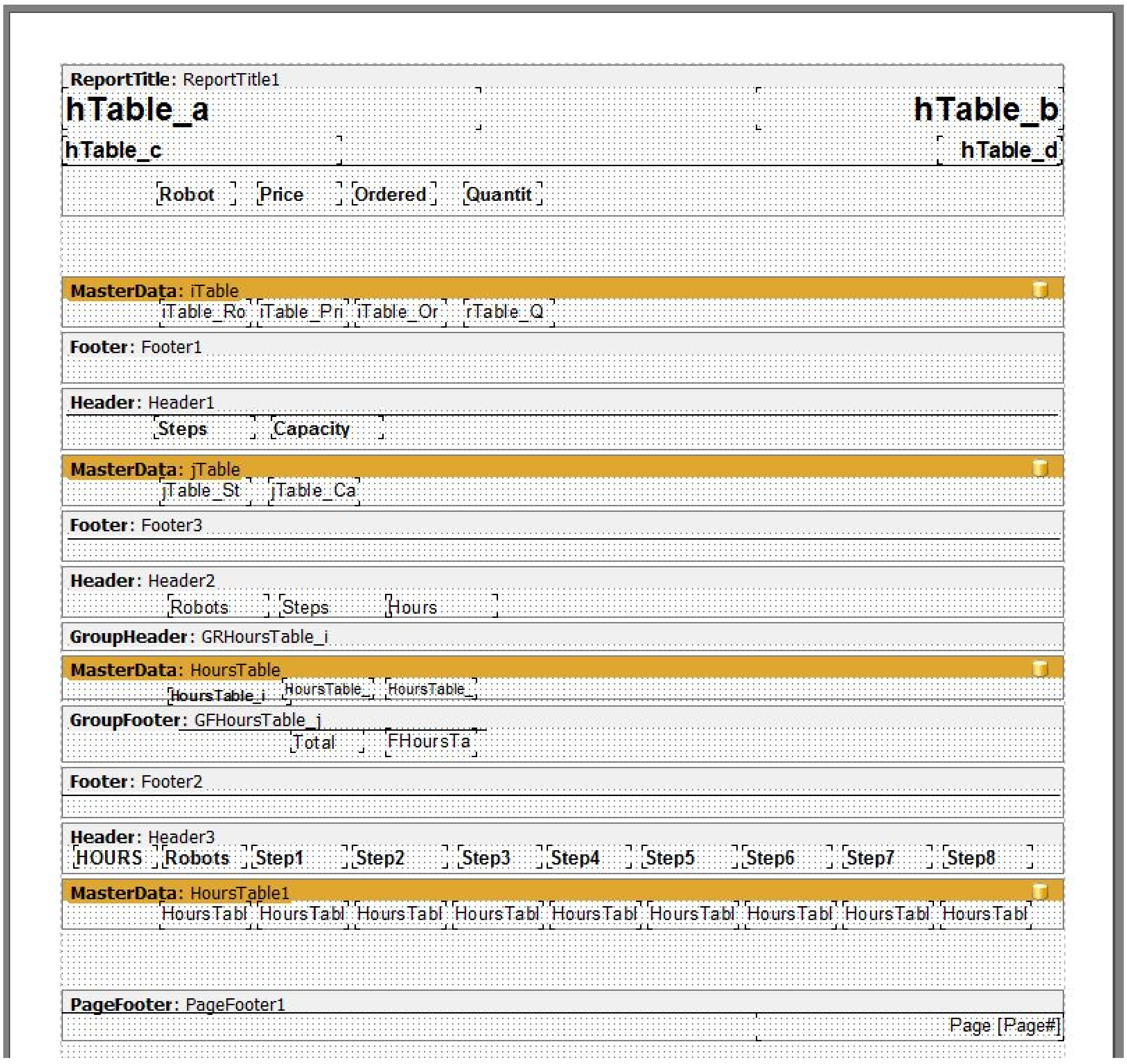
1. Replace `*.pdf` by `*.rtf`, and run the model. What happens?

2. Open the template file `tutor08c1.fr3` and modify something. Run again.

Answers

1. A RTF-file is generated that can be opened by MS Word (or LibreOffice Writer).

2. Open the template `tutor08c1.fr3` (use lplw.exe menu item <View/Open Report Designer>). In the designer click on the memo `tutor08c1_d`. On the property page change the `DisplayFormat` to Date/Time. Save and exit the template. Run the model again. The modified report is generated to file `tutor08c1.pdf` .



ReportTitle: ReportTitle1
hTable_a hTable_b
hTable_c hTable_d
Robot Price Ordered Quantit

MasterData: iTable
iTable_Ro iTable_Pri iTable_Or rTable_Q

Footer: Footer1

Header: Header1
Steps Capacity

MasterData: jTable
jTable_St jTable_Ca

Footer: Footer3

Header: Header2
Robots Steps Hours

GroupHeader: GRHoursTable_i

MasterData: HoursTable
HoursTable_i HoursTable_ HoursTable_

GroupFooter: GFHoursTable_j
Total FHoursTa

Footer: Footer2

Header: Header3
HOURS Robots Step1 Step2 Step3 Step4 Step5 Step6 Step7 Step8

MasterData: HoursTable1
HoursTabl HoursTabl HoursTabl HoursTabl HoursTabl HoursTabl HoursTabl HoursTabl HoursTabl

PageFooter: PageFooter1
Page [Page#]

Figure 9: Fast Report Template Page

16 Writing With Formatted Masks (tutor08d)

—- Run LPL Code , HTML Document –

Problem: This is the same model as `tutor07.lpl`, except that we use a unit cost per Hours production and the profit is maximized However, now we use a more complicated masks for model output using the `Write` statement.

Listing 16: The Complete Model implemented in LPL [2]

```
model TUTOR08d "Writing With Formatted Masks";
   /*$I 'tutor.inc' */  -- read the data
  parameter CpH := 5 "Cost per hour";
  --Price{i}:=2*Price;
  variable Robots{i};
  constraint
    Steps{j}: sum{i} Hours * Robots <= Capacity;
    Order{i}:   Robots  >=  Ordered;
  maximize profit: sum{i} Price*Robots -sum{i,j} CpH*Hours*Robots;
  Write('␣OUTPUT␣of␣the␣TUTOR08d␣model\n␣\
␣␣␣␣␣␣␣␣␣␣----------------------------\n␣\
␣␣␣␣␣␣␣␣␣␣type␣of␣␣␣number␣of␣␣␣␣already␣␣␣␣␣Price␣␣␣␣Cost\n␣\
␣␣␣␣␣␣␣␣␣␣␣robot␣␣␣␣␣␣robot␣␣␣␣␣␣ordered␣␣␣␣␣␣unit␣␣␣␣␣unit\n␣\
␣␣␣␣␣␣␣␣␣␣-------------------------------------------------\n');
  Write{i}('␣␣%7s␣␣␣␣␣%4d␣␣␣␣␣␣␣␣␣%3d␣␣␣␣␣␣␣␣%3d␣␣␣␣␣␣%3d␣␣␣%15s\n',
        i,Robots,Ordered,Price, sum {j}CpH*Hours,
         if (Price<sum {j}CpH*Hours,'loosing␣money','ok'));
  Write('␣---------------------------------------------------\n\n␣␣␣␣\
␣␣␣␣␣␣␣␣␣␣Total␣of␣revenue␣:␣␣␣␣␣␣␣␣␣␣␣%9.2f\n␣␣␣␣\
␣␣␣␣␣␣␣␣␣␣Total␣of␣costs␣␣␣:␣␣␣␣␣␣␣␣␣␣␣%9.2f\n␣␣␣␣\
␣␣␣␣␣␣␣␣␣␣Total␣of␣profit␣␣:␣␣␣␣␣␣␣␣␣␣␣%9.2f\n',
            sum {i}Price*Robots,
            sum {i,j}CpH*Hours*Robots,
            sum {i}Price*Robots - sum {i,j}CpH*Hours*Robots);
end
```

LPL Modeling Steps

1. The `Write` statement can be used to print formatted text defined by the user distrubuted over several lines. The mask contains line feed chars (`\n`) to write a new line. At the end of a mask line we use a back-slash char \ to remove the following line feed and the beginning blank chars on the following line within the mask.

2. The `Write` expression instructs LPL how to fill this mask.

Note: The model tutor08e[15] lists all format specifiers that can be used in LPL.

Questions

1. Double the `Price` vector and see what happens.

Answers

1. To double the prices, one only needs to add an instruction before the declaration of the variables, for example. Hence, add the line

[15] https://lpl.matmod.ch/lpl/Solver.jsp?name=/tutor08e

```
    Price{i} := 2*Price;
```

Then run the model again. The result shows that we are loosing money on `Robot4` and `Robot5` only now.

17 Write-Format Examples (tutor08e)

—- Run LPL Code , HTML Document –

Problem: This model shows all output formatting options with the `Write` statement as explained in the reference manual of LPL in Chap. 9.2.2. These options correspond to the specifications which are used by other languages such as Java.

Note that these specifiers can also be used in the `Format` function that returns the formatted string. An example can be seen at the end of the LPL code.

Listing 17: The Complete Model implemented in LPL [2]

```
model TUTOR08e "Write-Format Examples";
  parameter a := 1234.56;
    integer i := 1234;
    string  b := 'abcdefg';
    date    d := @2004-11-13T10:30:15;
  Write('d␣(integer)␣␣␣␣␣␣␣:␣%d\n'      ,-i);
  Write('u␣(unsigned)␣␣␣␣␣␣:␣%u\n'      , i);
  Write('h␣(hexadeci)␣␣␣␣␣␣:␣%x\n'      , i);
  Write('o␣(octal)␣␣␣␣␣␣␣␣␣:␣%o\n'      , i);
  Write('f␣(float)␣␣␣␣␣␣␣␣␣:␣%f\n'      , a);
  Write('f␣(width␣5,␣1␣dec):␣%5.1f\n'   , a);
  Write('f␣(right␣adjusted):␣%-15.3f\n', a);
  Write('f␣(left␣adjusted)␣:␣%15.3f\n'  , a);
  Write('e␣(e-notation)␣␣␣␣:␣%.2e\n'    , a);
  Write('e␣(e-notation)␣␣␣␣:␣%16.7e\n'  , a);
  Write('g␣(same␣as␣e␣or␣f):␣%16.7g\n'  , a);
  Write('n␣(with␣1000␣sep)␣:␣%n\n'      , a);
  Write('n␣(4␣decimals)␣␣␣␣:␣%.4n\n'    , a);
  Write('n␣(1␣decimal)␣␣␣␣␣:␣%7.1n\n'   , a);
  Write('m␣(currency)␣␣␣␣␣␣:␣%m\n'      , a);
  Write('m␣(7,3␣width)␣␣␣␣␣:␣%7.3m\n'   , a);
  Write('b␣(boolean␣true)␣␣:␣%b\n'      , 1);
  Write('b␣(boolean␣false)␣:␣%b\n'      , 0);
  Write('s␣(string)␣␣␣␣␣␣␣␣:␣%s\n'      , b);
  Write('s␣(width␣10,5)␣␣␣␣:␣%10.5s\n'  , b);
  Write('z␣(fraction)␣␣␣␣␣␣:␣%z\n'      , .75);
  Write('z␣(fraction)␣␣␣␣␣␣:␣%z\n'      , 6.125);
  Write('z␣(fraction)␣␣␣␣␣␣:␣%z\n'      , 6.875);
  Write('\n--␣date␣now--\n');
  -- All date/time format specifiers
  Write('tH␣(Hour)␣␣:␣%tH\n' , d);
  Write('tk␣(Hour)␣␣:␣%tk\n' , d);
  Write('tI␣(Hour)␣␣:␣%tI\n' , d);
  Write('tl␣(Hour)␣␣:␣%tl\n' , d);
  Write('tM␣(Min)␣␣␣:␣%tM\n' , d);
  Write('tS␣(Secs)␣␣:␣%tS\n' , d);
  Write('tL␣(mSecs)␣:␣%tL\n' , d);
  Write('tp␣(ampm)␣␣:␣%tp\n' , d);
  Write('tB␣(month)␣:␣%tB\n' , d);
  Write('tb␣(month)␣:␣%tb\n' , d);
  Write('th␣(month)␣:␣%th\n' , d);
  Write('tm␣(month)␣:␣%tm\n' , d);
  Write('tA␣(wday)␣␣:␣%tA\n' , d);
  Write('ta␣(wday)␣␣:␣%ta\n' , d);
  Write('tY␣(year)␣␣:␣%tY\n' , d);
```

```
    Write('ty␣(year)␣␣:␣%ty\n' , d);
    Write('td␣(day)␣␣␣:␣%td\n' , d);
    Write('te␣(day)␣␣␣:␣%te\n' , d);
    Write('tR␣(h:m)␣␣␣:␣%tR\n' , d);
    Write('tT␣(h:m:s)␣:␣%tT\n' , d);
    Write('tr␣(h:m:s)␣:␣%tr\n' , d);
    Write('tD␣(y-m-d)␣:␣%tD\n' , d);
    Write('tR␣(y-m-d)␣:␣%tF\n' , d);
    Write('tc␣(all)␣␣␣:␣%tc\n' , d);
    string parameter mydate:=Format('DAY:(%tc)␣␣TIME:(%tT)', d,d);
    Write('\nThe␣complete␣date␣is:␣%s\n', mydate);
    set s:=1..5;
    Write('\n%s', {s} Format('%2s^2=%1d␣␣␣',s,s^2));
  end
```

The Format function is especially useful in combination within a Write. The last Write in code shows how. The Format function returns a string. This string is concatenated over all elements of s and returned as parameter to the Write, that outputs the whole as a string.

18 Submodels (output locally only) (tutor09)

–– Run LPL Code , HTML Document –

Problem: This model show how to arrange a model into several submodels that can be called by the main model.

Listing 18: The Complete Model implemented in LPL [2]

```
model TUTOR09 "Submodels (output locally only)";
  mydata;
  writedata;
  ClearData(TUTOR09);
  readdata;
  set   i; j;
  parameter  a{i};  b{j};
  string parameter s{i};
  ------------------------------------------------
  model mydata "define some data";
    SetRandomSeed(1);
    i      := 1..10;
    a{i}   := Trunc(Rnd(1,10));
    s{i} := ['AA','BB','CC','DD','EE','FF','GG','HH','II','JJ'];
    j      := 1..11;
    b{j}   := j^2;
  end
  model writedata "write the data to a file";
    Write('tutor09out.txt','--␣Data␣generated␣by␣LPL\n\n');
    Write('Table␣1␣--␣(i␣a␣s)\n');
    Write{i}('␣%3s␣%3d␣%4s\n', i,a,s);
    Write('\nTable␣2␣--␣(j␣b)\n');
    Write{j}('␣%3s␣%4d\n', j,b);
  end
  model readdata;
    Read('tutor09out.txt,%1:Table');
    Read{i}('%1', i,a,s);
    Read{j}('%2', j,b);
  end
end
```

LPL Modeling Steps

The model contains three submodels, called `mydata`, `writedata`, and `readdata`. The main model (`TUTOR09`) calls them in this order. It also calls a function `Cleardata(TUTOR09)` that clears all the data defined in the main model.

The main model also declares some tables: a set `i` and `j`, two numerical vectors `a{i}` and `b{j}`, and a string vector `s[i]`.

The three submodel can access and modify the 'global" data defined in the main model. The first submodel `mydata` sets the random seed and fills all the data vectors. Note that a table assignment such as `*s{i} := [ ... ]` must begin with a star, to specify that the data are assigned through an internal table an not through an expression.

The submodel `writedata` write the data to a file subdivided into two blocks.

The submodel `readdata` reads these data again from that file.

The generated file is as follows:

Questions

1. Replace `SetRandomSeed(1)` by `SetRandomSeed(2)`. What happens?

Answers

1. A different data set is now generated for `a{i}`, since this the only vector generated randomly, using the function `Rnd` that returns a uniormly distributed random number.

19 Sparse Tables (tutor10)

—- Run LPL Code , HTML Document –

Problem: This model shows sparse data tables based on expressions.

Listing 19: The Complete Model implemented in LPL [2]

```
model TUTOR10 "Sparse Tables";
  set   i; j;
  parameter  Ordered{i}; Price{i}; Capacity{j};
      Hours{i,j | i<>10 or j<>4};
  variable   Robots{i | Ordered>6};
  -- read the data here
  Read('tutor.txt,%1:Table');
  Read{i}('%1', i,Price,Ordered);
  Read{j}('%2', j,Capacity);
  Read{i}('%3', i, {j}Hours);
  constraint
    Steps{j}: sum{i} Hours * Robots <= Capacity;
    --Steps{j|j<=3}: sum{i} Hours * Robots <= Capacity;
    Order{i}:   Robots  >=  Ordered;
  maximize profit: sum{i} Price * Robots;
  Writep(profit,Robots,Hours);
end
```

LPL Modeling Steps

Sparsity is one of the key concept in LPL. Multidimensional tables are stored and handeled in a sparse way, which means that only a subset of the Cartesian Product of table elements are stored. A four-dimensional table of only 1000 elements in each dimension goes beyond every memory allocation, if considered as full table. It is, therefore, essential to deal with sparsity. LPL has efficient ways to deal with it.

1. An index-set can be limited by an arbitrary expression, for example. The following declaration

   ```
       variable Robots{i | Ordered > 6};
   ```

   means that a variable `Robots` is declared for every element in `i` such that the order is larger then 6. In mathematical notation:

   $$x_i, \quad \forall\{i|i \in I = \{1\ldots n\}\ ,\ O_i > 6\}$$

   In our case it means that the three variables `Robot[4]`, `Robot[5]` and `Robot[10]` are discarded from the model.

2. A tuple list `{i,j}` also represents a set and can, therefore also be limited by expressions. Hence, the declaration:

   ```
       Hours{i,j | i<>10 or j<>4};
   ```

   means that there exists a value within table `Hours` for each combination of $(i, j) \in I \times J$ such that $i \neq 10$ or $j \neq 4$.

3. The condition `i<>10 or j<>4` excludes one tuple: 'declare `Hours` for all tuples `(i,j)` except for $i = 10$ and $j = 4$. (The number 10 and 4 indicate the positions of the elements within the sets. Sets are considered always as ordered in LPL.) (The condition could also be written as `~(i=10 and j=4)`. If the expression evaluates to zero, it is interpreted as 'tuple does not exist', else as 'tuple does exist'.

4. Any index-list may be followed by a condition. Suppose we want the maximizing function only summed up over all Robots yielding a price greater than 100. We could then write the maximizing function as follows:

```
maximize profit: sum{i | Price>100} Price * Robots;
```

Questions

1. How does the model change, if one modifies the condition `Ordered>5` to `Ordered>6` in the variable declaration?

2. Change the constraint definition `Steps{j}:...` to `Steps{j|j<=3}....` Analyse the solution.

Answers

1. The variable `Robot4` is removed from the model. Note that it is enough to remove it at the declaration. Summation over "all" `i` then discards them automatically too.

2. The solution did not change! Why? This is because all the constraints `Steps[4]` to `Steps[8]` are not "tight", that is, they are not at their maximal capacity.

20 Predefined Functions (tutor11)

—- Run LPL Code , HTML Document –

Problem: In this model, all data are generated by a random generator. The model shows and explains several functions used in LPL.

Listing 20: The Complete Model implemented in LPL [2]

```
model TUTOR11 "Predefined Functions";
  SetRandomSeed(1);
  set i := [1..10]; j := [1..8];
  parameter
    Ordered{i}  := Trunc(Rnd(3,20));
    Price{i}    := Trunc(Rnd(100,200));
    Capacity{j} := Trunc(Rnd(3000,5000));
    Hours{i,j}  := Trunc(Rnd(0,9));
  variable   Robots{i};
  constraint
    Steps{j}: sum{i} Hours * Robots <= Capacity;
    Order{i}:   Robots  >=  Ordered;
  maximize   profit: sum{i} Max(Price,100) * Robots;
    /* another formulation is:
       profit: sum(i) If(Price<100,100,Price) * Robots; */
  Writep(profit,Robots);
end
```

LPL Modeling Steps

All data are generated by a random generation procedure.

1. The seed of the random generator is initialized by `SetRandomSeed(1)`.
2. The function `Rnd(a,b)` generates a uniform number between `a` and `b`. The function `Trunc(a)` truncates the number `a` to an integer.
3. Other similar functions are `Ceil(x)` (returns the largest integer smaller than `x`), `Round(x)` (returns the rounded integer of `x`), `Sin(x)` (returns the sinus of `x`), `Abs(x)`, `Tan()`, `Cos()`, `Log()`, and many others,
4. The `Max` function returns the larger of the two operands. Hence, `Max(12,10` means `12`. The *smaller of*-function is `Min`.
5. The `if` function takes at least two arguments. The first is a condition. The second argument is evaluated if the condition evaluates to non-zero (true), else the third argument is evaluated. If the third argument is missing, zero is assumed. Example:

   the statement `a := if(3=4 , 4 , 3);` will assign 3 to `a`.
6. The `if` can have more than three arguments which is interpreted like a `switch` statement in C. For example:

   ```
   If(a,b,c,d,e,f,g)
   ```

   means:“if `a` is true (non-zero) return `b`, else if `c` is true return `d` else if `e` is true return `f`, else return `g`. Of course, all the parameters `a` to `g` can be arbitrary expressions.

Questions

1. How can the prices be generated as a normal distributed vector with mean 100 and a deviation of 20?
2. Sum all prices that are larger than 120 and assign the value to a new parameter.

Answers

1. Use the function `Rndn(m,d)`, it returns a normally distributed random number with mean `m` and standard deviation `d`. The statement is:

   ```
    Price{i} := Rndn(100,20);
   ```

2. The statement is as follows:

   ```
   parameter A := sum{i|Price>120} Price;
   ```

   There is another way to express the same expression as follows:

   ```
   parameter A := sum{i} if(Price>120,Price);
   ```

   Note the `if` with two parameters returns zero as default if the condition is false (equal to zero).

21 Index Operators (tutor12)

—- Run LPL Code , HTML Document –

Problem: Indexed operators are a powerful mean to concisely write large expressions. In mathematical notation, for example, one can write

$$\sum_{i \in \{1...1000\}} x_i$$

which is a shortcut for

$$x_1 + x_2 + x_3 + \ldots + x_{999} + x_{1000}$$

The operator $\sum$ is called *indexed operator* for summig up over a set. There exist other such operators, for example max (that returns the largest element in a list) or argmin (which returns the list position of the minimal element). In LPL, these operators can be used in the same way.

Listing 21: The Complete Model implemented in LPL [2]

```
model TUTOR12 "Index Operators";
  /*$I 'tutor.inc' */
  set i1{i} :=
    sum{j}Hours>24 and (forall{j}(Hours<8) or exist{j}(Hours=0));
  variable Robots{i|i1};
  constraint
    Steps{j}: sum{i} Hours*Robots <= Capacity;
    Order{i}:   Robots  >=  Ordered;
  maximize profit: sum{i} Price*Robots;
  parameter a:=max{j} min{i} Hours;
  Writep(profit,Robots);
  Write{i|sum{j}Hours>24}('%s␣uses␣more␣than␣24h␣to␣be␣produced\n',i);
  Write{i|forall{j}(Hours<8)}('%s␣used␣less␣then␣8h␣in␣each␣step\n',i);
  Write{i|exist{j}(Hours=0)}('%s␣does␣not␣use␣some␣steps\n',i);
  Write{j|atleast(5){i}(Hours>5)}('In␣%s␣at␣least␣5␣robots␣use␣more␣
     than␣7h\n',j);
end
```

LPL Modeling Steps

In various locations of the model we use index operators.

1. The expression `sum{i} Price*Robots`, used in previous models, means the same as $\sum_i Price_i \cdot Robots_i$.

2. Two other operators are used: `forall` and `exist`. Let's explain the expression containing them.

3. The partial expression `sum{j} Hours[i,j]`, calculates the number of hours used to produce a Robot `i` ($\sum_j H_{i,j}$, $\forall i$). It returns a value for each i.

   The expression `sum{j} Hours{i,j] > 24` returns for each i true or false, depending on whether the sum is larger than 24. (This is the case for Robot 1, 3, 4, 5, 7, and 8, as can be verified with the first `Write` statement.

4. The expression `forall{j}(Hours<8)` returns true or false, depending on whether *all* values of $Hours_{i,j}$ for a particular i are smaller than 8. (This is the case for Robot 3, 7, 9, and 10.)

5. The expression `exist{j}(Hours=0)` returns true or false, depending on whether there exists a value of $H_{i,j}$ for a particular i that is zero. This is the case for all robots except 10.)

6. These three expressions combined form a subset defined as `i1{i}`. Hence LPL's way to define subsets is indexing over a set

```
    set i1{i};
```

7. The operator `atleast` is another (Boolean) indexed operator, that returns true if at least a given number of Boolean expressions are true.

Questions

1. What is returned by: `max{i} Price` ?

2. What is returned by: `argmin{i} Price` ?

3. What means: `max{i} min{j} Hours` ?

4. What means: `atleast(5){i,j} (Hours>7)` ?

Answers

1. 400. This is the largest value in the price list. Robot9 costs 400.

2. 4. The smallest price value (50) is at the fourth position within the price list (Robot4 costs 50).

3. It means "choose the smallest value in each column `j` of the matrix `Hours{i,j}` and from the obtained list choose the largest value". It is 4, because in step 1 the smallest value is 4.

4. It means that "at least five values in the table `Hours` are greater than 7". True or false? This expression returns true (1).

22 Expression Evaluation (tutor13)

—- Run LPL Code , HTML Document –

Problem: An LPL model does not need to be an optimisation model. Any sequence of declarations and instructions can be defined.

Listing 22: The Complete Model implemented in LPL [2]

```
model TUTOR13 "Expression Evaluation";
  /*$I 'tutor.inc' */
  variable  Robots{i};
  constraint
    Steps{j}: sum{i} Hours*Robots <= Capacity;
    Order{i}: Robots  >=  Ordered;
    C: Robots[9]=420;
  maximize revenue: sum{i} Price*Robots;
  parameter
    d{j}:=sum{i}Ordered*Hours;
    rCapa{j}:=Capacity-d;
    MaxR{i} := Floor(min{j|Hours} rCapa/Hours);
  Write('␣␣Robot␣␣␣␣␣Ordered␣␣␣MaxR␣␣␣␣␣␣Total\n␣␣\
␣␣␣␣␣␣␣␣␣␣␣-----------------------------------\n');
  Write{i}('␣␣%-7s␣␣␣␣%3d␣␣␣␣␣␣␣%3d␣␣␣␣␣␣%4d\n',
     i,Ordered,MaxR,MaxR+Ordered);
  Writep(revenue,Robots,Price,Ordered,Capacity,rCapa,Hours);
end
```

LPL Modeling Steps

The data are read from the file `tutor.inc`.

1. `min` and `max` are two index-operators which returns the minimum or maximum of a list of expressions.

2. The expression `sum{i}Ordered*Hours` returns for each step `j` the used capacity of the already `Ordered` robots. `rCapa{j}` is then the capacity that remains for the optimized production.

3. The parameter `MaxR{i}` calculates for each robot type `i` the maximally producible quantity of robots with respect to the capacities that is different from zero in the table `Hours` over `j`. Note that the condition "different from zero" is necessary in the definition otherwise it will be simply zero.

Questions

1. Verify by solving the model that one cannot produce more than 520 units of robots of type 9.

2. Robot type 9 has the highest price. So it makes sense to produce the maximum possible – that is 520 – of it. What is the revenue, if one fixes the production of robot type 9 to 520?

3. Why does the previous fixing of variable 9 not generate the maximal revenue?

4. Simplify the expression `max{i} max{j} Hours[i,j]`.

Answers

1. Add a the following constraint

   ```
   	constraint C: Robots[9]>=521;
   ```

   and solve the model. The model is infeasible (see message left below on the lplw screen). The model has no solution because the capacity limits the production of robot of type 9 to 520.

2. The revenue is 222950. This is less than the maximally possible revenue of 232920.59.

3. The reason is not so easy to see. A production of 520 units of robots 9 would use all capacity in step 3. However, Robot 2 – which has the second highest price – does not use that much capacity in step 3. So may be it would be worthwhile to leave capacity for robot 2 in order to get out more of the remaining capacities. That is indeed the case: We must reduce the production of Robot 9 to 420 to allow more production for robot 2, in order to increase revenue.

4. The two indices can be combined into one, One can use the second name `H`, and the index-list `[i,j]` is not necessary. Hence, it is: `max{i,j} Hours`.

23 Goal Programming (tutor14)

—- Run LPL Code , HTML Document –

Problem: LPL can be used to model soft constraints using goal programming. The idea is – in our small example – that we do not know exactly the capacity of a process. That is, we want to integrate uncertainty, see explanation below.

Listing 23: The Complete Model implemented in LPL [2]

```
model TUTOR14 "Goal Programming";
  variable Marie; Jules;
     PTesting; Nrevenue; Prevenue;  --slack variables
  parameter dP:=18500  "desired revenue";
  constraint
    Component:  5*Marie + 5*Jules  <=  350;
    Mounting:   4*Marie + 8*Jules   <=  480;
    Testing:    6*Marie + 2*Jules -PTesting  <=  300;
    Order1:       Marie               >=  20;
    Order2:                 Jules   >=  15;
    revenue: 300*Marie + 200*Jules -Prevenue+Nrevenue = dP;
    --TestBound: PTesting=10;
  minimize deviation: Nrevenue + PTesting + Prevenue;
  --minimize deviation: Nrevenue + 200*PTesting + Prevenue;
  Writep(deviation,Marie,Jules,Nrevenue,PTesting);
end
```

LPL Modeling Steps

Let's return to the simplest model `tutor02` with one exception.

1. Suppose we do not know exactly the capacity of `Testing`, but we know that it is at least 300.
2. Furthermore, we will be happy with a revenue of "about" 18500.
3. This problem would be infeasible as we know from problem tutor02 (since the maximizing revenue is 18000).
4. Hence, we add a positive slack variable (`PTesting`) in order to see how much the capacity of `Testing` has to be expanded exceeding 300 to attain our objective.
5. In the revenue constraint, we add two slack variables to cover a positive or negative deviation (`Prevenue` and `Nrevenue`).
6. We do not maximize revenue, as before, but we minimize a deviation measure.
7. The deviation of the positive `PTesting` and the negative `Nrevenue` slacks is minimized.
8. As can be seen from the solution, the optimum 18500 can be attained if we allow the `Testing` constraint to exceed its capacity of 300 by 20 units.
9. If we add an upper bound of 10 to PTesting (`TestBound`)then we can still attain a revenue of 18250.

Questions

1. Modify the value of `dP` from `18500` to `20500` by steps of 200. What do you notice when solving each time?
2. Change the model in a way to make `PTesting` as small as possible.

Answers

1. The revenue cannot be augmented over 19500, even if the "desired revenue" `dP` gets higher. This is because of the `Components` constraint is at 100% percent of its capacity and the number of `Jules` cannot be lowered below 15, because of the bound imposed by `Order2` and consequently the number of `Marie` cannot be higher than 55. The next model `tutor15.lpl` shows how these calculations can be implemented in a single LPL model code.
2. We could replace the minimizing function to `minimize dev: PTesting;`. Another way is to leave the minimizing function, but to impose a higher "penalty" to the term `PTesting` by adding a factor, for example: `200*PTesting`. Penalising is a general method to formulate multiple objectives.

24 Loop Programming (tutor15)

—- Run LPL Code , HTML Document –

Problem: This model is the answer to the first question in the previous model tutor14[16]. How to implement a sequence of optimisations in a single model? We call this also “parameterized optimisation”.

Listing 24: The Complete Model implemented in LPL [2]

```
model TUTOR15 "Loop Programming";
  variable Marie; Jules;
    PTesting; Nrevenue; Prevenue; --slack variables
  parameter dP:=18500  "desired revenue";
  constraint
    Component:  5*Marie + 5*Jules   <=  350;
    Mounting:   4*Marie + 8*Jules   <=  480;
    Testing:    6*Marie + 2*Jules -PTesting  <=  300;
    Order1:       Marie             >=  20;
    Order2:                 Jules   >=  15;
    revenue: 300*Marie + 200*Jules -Prevenue+Nrevenue = dP;
    --TestBound: PTesting=10;
  --- loop
  set i:=1..10;
  parameter dP1{i}; Marie1{i}; Jules1{i}; revenue1{i};
  for{i} do
    minimize deviation: Nrevenue + PTesting + Prevenue;
    dP := dP+200;
    dP1[i]:=dP; Marie1[i]:=Marie;
    Jules1[i]:=Jules; revenue1[i]:=300*Marie + 200*Jules;
  end;
  Write('␣␣dP␣␣␣␣␣␣␣Marie␣␣␣␣Jules␣␣␣␣revenue\n␣␣\
␣␣␣␣␣␣␣␣␣␣␣----------------------------------\n');
  Write{i}('␣␣%5d␣␣␣␣␣%3d␣␣␣␣␣␣%3d␣␣␣␣␣␣%5d\n',
     dP1,Marie1,Jules1,revenue1);
end
```

LPL Modeling Steps

The only modification takes place at the `minimize` function. It is embedded in a loop: The model is solved 10 times each time with a different parameter `dP`.

1. First we add a set `i`. The cardinality of this set (here 10) is the number of optimisations.
2. `for` is another index operator that can be used to loop through a set, in our case over `i`.
3. Within the loop, the deviation is minimized and the parameter `dP` is increased in steps of 200.
4. Certain data are stored for later output and the loop is repeated.

Questions

1. What happens if the capacity of `Mounting` is extended? Can we make more revenue?

[16]https://lpl.matmod.ch/lpl/Solver.jsp?name=/tutor14

2. We would like to rise `dP` by 2% within the loop. How can this be done?

Answers

1. Nothing happens, the revenue is the same. This is because `Mounting` is never the limiting constraint. We would need to extend the `Component` capacity.
2. Replace the instruction `dP:=dP+200;` with `dP:=Trunc(1.02*dP);` and run the model again.

25 Logical Constraints (tutor16)

—- Run LPL Code , HTML Document –

Problem: We include some logical constraints to our well know basic model tutor06[17].

Listing 25: The Complete Model implemented in LPL [2]

```
model TUTOR16 "Logical Constraints";
  /*$I 'tutor.inc' */ -- a data file is included here
  integer variable  Robots{i}  [0..500];
  constraint
    Steps{j}: sum{i} Hours*Robots <= Capacity;
    Order{i}: Robots  >=  Ordered;
    Log1:     Robots[2]>=130 -> Robots[6]>=30;
    Log2:     atleast(5){i} (Robots[i]>=21);
    Log3:     Robots[3]>=10 and Robots[4]>=15;
  maximize revenue: sum{i} Price*Robots;
  Writep(revenue,Robots);
end
```

LPL Modeling Steps

The basic model (without the logical constraint Log1, Log2, Log3, and Log4) has the following solution:

```
revenue = 232850.00
Robot1 Robot2 Robot3 Robot4 Robot5 Robot6 Robot7 Robot8 Robot9 Robot10
  20     266      7      6      5      8      9      8     420      5
```

LPL allows to add Boolean operators within constraints. LPL translates these constraints automatically into mathematical linear constraints by introducing binary (0-1) variables. These feature allow the modeler to formulated certain requirements directly in a logical context.

Example: Suppose the following condition is required; “If the number of robot type 2 is larger or equal than 130 then at least 30 of robot type 6 should also be manufactured”. This is a logical implication formulated as:

$$Robot_2 \geq 130 \rightarrow Robot_6 \geq 30$$

In LPL, this can be formulated in a very similar way:

```
  constraint Log1: Robots[2]>=130 -> Robots[6]>=30;
```

Adding this constraint to our basic model produces the following solution:

```
revenue = 227450.00
 Robot1 Robot2 Robot3 Robot4 Robot5 Robot6 Robot7 Robot8 Robot9 Robot10
   20     254      7      6      5     30      9      8     407      5
```

We can verify that the condition is fulfilled: Robots[2] *is* larger than 130, therefore Robots[6] is at least 30 (which is the case in this solution). Of course, the revenue drop to 227450. That is the “price” the fulfill the condition.

LPL also allows one to use logical indexed operators. Suppose, we want to impose the following condition: “For at least 5 different type of robots, we need to manufacture a minimal quantity of 21”. In LPL, this can directly be formulated as follows:

[17]https://lpl.matmod.ch/lpl/Solver.jsp?name=/tutor06

```
constraint Log2: atleast(5){i}(Robots[i]>=21);
```

Adding this constraint to the previous version (with `Log1` produces the following optimal solution:

```
revenue = 225550.00
Robot1 Robot2 Robot3 Robot4 Robot5 Robot6 Robot7 Robot8 Robot9 Robot10
  21    231     7      6      5      30     9      8     405     21
```

One can easily verify that the condition `Log2` also holds: At least 5 different robot types are manufactured at minimal quantity of 21. At the cost of a further reduction in the revenue.

Let's add a third condition:"at least 10 units of robot 3 and at least 15 of robot 4 must be manufactured". The condition in LPL is:

```
constraint Log3: Robots[3]>=10 and Robots[4]>=15;
```

The optimal solution with the three first conditions becomes:

```
revenue = 221000.00
Robot1 Robot2 Robot3 Robot4 Robot5 Robot6 Robot7 Robot8 Robot9 Robot10
  21    129     10     15     5      8      9      21    445     21
```

Note also that the condition `Log1` is not used anymore: Because the number of robots of type 2 drops to 129 (which is smaller than 130) there is no need that the number of robot of type 6 is at least 30.

The condition `Log3` can easily be implemented without any logical operators: it says simply that `Robots[3]` is bounded from below to 10 and `Robots[4]` has a lower bound of 15. The condition `Log3` just emphasises simply that the Boolean `and` operator *is* the default operator of a list of constraint.

Questions

1. Formulate the constraint that none of the Robots type can exceed 100 pieces.
2. Formulate the condition: "If the number of robot type 1 is larger or equal than 10 then the number of robot type 5 must be larger or equal than 12 or the number of robot type 7 must be larger or equal than 10."

Answers

1. A simple version is to set a lower bound for every variables as follows:

   ```
   constraint Log4{i}: Robots<=100;
   ```

   Another way to formulate it as a logical condition is:

   ```
   constraint Log4: nor{i} (Robots>=100);
   ```

   `nor` is another index operator which formulate the logical "none". Do not confound it with the operator `nand`. The operator `nand` – another operator in LPL – means "not all". By the way, LPL translates the constraint `Log4` automatically into ordinary variable bounds (see EQU-file).

2. The constraint in LPL is as follows:

   ```
   constraint Log5: Robots[1]>=10 -> Robots[5]>=12 or Robots
      [7]>=10;
   ```

26 Some Basic Expressions / Writes (learn01)

–– Run LPL Code , HTML Document –

Problem: This code show some simple operations and functions as well as Write formatted output using Write().

Listing 26: The Complete Model implemented in LPL [2]

```
model learn01 "Some Basic Expressions / Writes";
   parameter
     x := 5^7                 "power operator";
     y := 1=1 or 0 and ~0 "Boolean expression";
     z1  := Sin(34)           "Sinus";
     z2  := Log(1000)         "log of base e";
     z3  := Max(34,6)         "return the larger";
     z3a:= Min(34,6)          "return the smaller";
     z4  := 45/67*976         "arithmetic";
     z5  := 1<2               "Boolean";
     z6  := x                 "assignment";
     z7  := x/56              "item";
     z8  := y                 "Boolean assign";
   string parameter
     z9  := 'abcd' & 5        "String expression (concat)";
     zA  := '' & Sin(x)       "cast double to string";
   date parameter zB :=@2018-12-08 "a date";
   Write('5^7␣␣␣␣␣␣␣␣␣␣␣=␣%5d\n\
␣␣␣␣␣␣␣␣␣␣1␣or␣0␣and␣~0␣=␣%1d\n\
␣␣␣␣␣␣␣␣␣␣Sin(34)␣␣␣␣␣␣␣=␣%6.4f\n\
␣␣␣␣␣␣␣␣␣␣Log(1000)␣␣␣␣␣=␣%8.6f\n\
␣␣␣␣␣␣␣␣␣␣Max(34,6)␣␣␣␣␣=␣%2d\n\
␣␣␣␣␣␣␣␣␣␣Min(34,6)␣␣␣␣␣=␣%2d\n\
␣␣␣␣␣␣␣␣␣␣45/67*976␣␣␣␣␣=␣%7.3f\n\
␣␣␣␣␣␣␣␣␣␣1<2␣␣␣␣␣␣␣␣␣␣␣=␣%1d\n\
␣␣␣␣␣␣␣␣␣␣x␣␣␣␣␣␣␣␣␣␣␣␣␣=␣%5d\n\
␣␣␣␣␣␣␣␣␣␣x/56␣␣␣␣␣␣␣␣␣␣=␣%8.4f\n\
␣␣␣␣␣␣␣␣␣␣y␣␣␣␣␣␣␣␣␣␣␣␣␣=␣%1d\n\
␣␣␣␣␣␣␣␣␣␣\'abcd\'␣&␣5␣␣␣␣=␣%s\n\
␣␣␣␣␣␣␣␣␣␣\'\'␣&␣Sin(x)␣␣␣=␣%s\n\
␣␣␣␣␣␣␣␣␣␣date␣as␣int␣␣␣=␣%d\n\
␣␣␣␣␣␣␣␣␣␣a␣date␣format␣=␣%tc\n'
           , x, y, z1,z2,z3,z3a,z4,z5,z6,z7,z8,z9,zA,zB,zB);
end
```

27 Basic Indexed Expressions/Tables (learn01a)

–– Run LPL Code , HTML Document –

Problem: This code show some simple operations and functions.

Listing 27: The Complete Model implemented in LPL [2]

```
model learn01a "Basic Indexed Expressions/Tables";
   --- Tables expressions
   set i := [I1 I2 I3 I4 I5 I6 I7];
       j := [J1 J2 J3 J4 J5 J6 J7 J8 J9];
       k := [K1 K2 K3 K4 K5 K6 K7 K8 K9 K10];
   parameter a{i,j} := i*j;
             b{i,j} := i^2+j;
             c{j,k} := Sin(j)+Log(k);
  ---- Matrix calculation
   parameter
     d{i,j} := a[i,j] + b[i,j] "matrix addition";
     e{i,k} := sum{j} a[i,j] * c[j,k] "matrix multiplication";
   Writep(d,e);
  ---- Random tables, sparse tables
   SetRandomSeed(10) "sets the random seed"
   parameter ra{i,j,k} := Rnd(10,20) "a 3-dimen. table";
     f{i,k} := sum{j} 10*ra[i,j,k] + if(e[i,k]>100, e[i,k] , 100);
     g{i,k|i<k} := 3  "sparse table";
   Writep(ra,f,g);
   ---- 6. the first 50 Fibonacci numbers
   set s := [ 1..50 ];
   parameter h{s}; H{s};;
   h[1]:=1 , h[2]:=2,
   {s|s>2} (h[s] := h[s-1]+h[s-2]);
   {s} (H := if(s=1,1,s=2,2,H[s-1]+H[s-2])) "the same";
   Write('-----FIBONACCI␣NUMBER␣1-50:\n');
   Write{s}( '%2s␣%13d␣%13d\n', s,h,H);
end
```

28 The Addm function (learn01b)

—- Run LPL Code , HTML Document –

Problem: The function `Addm()` adds an element dynamically to a set, affecting all involved tables as well.

Listing 28: The Complete Model implemented in LPL [2]

```
model learn01b "The Addm function";
   set i := [I1 I2 I3 I4 I5 I6 I7];
       j := [J1 J2 J3 J4 J5];
   parameter a{i,j} := i*j;
   Writep(i,a);
   Addm(i,'ABC') "add dynmaically an element to set i";
   Writep( a,i);
   {j} (a[#i,j]:=100+j) "fill the added slide";
   Writep(a,i);
   set k:=1..10;
   Addm(k,'HJK') "another example: add an element to set m";
end
```

29 Some Math and Boolean Functions (learn02)

–– Run LPL Code , HTML Document –

Problem: This model shows further operators and functions of LPL.

Listing 29: The Complete Model implemented in LPL [2]

```
model learn02 "Some Math and Boolean Functions";
  ---- 1) predefined functions in LPL
  set i :=  [1..6] ;
  parameter
     a{i}     := [ 8.99  8.01  -8.99  -8.01  8.5  -8.5 ];
     b{i}     := [ 1.8   -8     4.01   7.35  9    -1.3  ];
     c{i}     := [ 0     20     0      40    50    60   ];
     a1{i}    := Ceil(a);
     a2{i}    := Floor(a);
     a3{i}    := Trunc(a);
     a4{i}    := Abs(a);
     p{i}     := i;
     maxi{i} := Max(a,b); --which is larger?
     mini{i} := Min(a,b); --which is smaller?
     ab{i}    := Abs(if(Abs(a)>Abs(b) , a , b));
     ln_b{i} := Log(Abs(b));
     expo{i} := Exp(a/7);
     I_10_20{i} := Rnd(20,30); -- uniform distributed in the range
        [20,30]
     N_10_1{i}  := Rndn(10,1); -- normal distributed , mean=10, std=1
  Write('␣␣␣a␣␣␣␣␣␣b␣␣␣␣␣Ceil␣␣␣Floor␣␣Trunc␣␣␣Abs␣␣␣␣Pos␣␣␣␣Max␣␣␣␣Min
      ␣␣␣Abs␣␣␣␣\
␣␣␣␣␣␣Ln(b)␣␣Exp␣␣␣␣Rnd␣␣␣␣Rndn\n');
  Write{i}('%6.2f␣%6.2f␣%6.2f␣%6.2f␣%6.2f␣%6.2f␣%6.2f␣%6.2f␣%6.2f␣%6.2f
      ␣%6.2f␣\
␣␣␣␣%6.2f␣%6.2f␣%6.2f\n', a,b,a1,a2,a3,a4,p,maxi,mini,ab,ln_b,expo,
    I_10_20,N_10_1);
  ---- 2) Index-operators
  parameter
     su   := sum{i} b[i];     -- sum all b[i]
     pr   := prod{i} b[i];    -- product
     ma   := max{i} b[i];     -- the maximum
     mi   := min{i} b[i];     -- the minimum
     pma  := argmax{i} b[i]; -- position of the maximum
     pmi  := argmin{i} b[i]; -- position of the minimum
     ex   := exist{i} c[i];  -- is there any non-zero? (TRUE=1)
     fo1  := forall{i} c[i]; -- are there all non-zeroes? (FALSE=0)
     fo2  := forall{i} b[i];
     atl3:= atleast(3){i} c[i];     -- are at least 3 non-zero? (TRUE=1)
     atl4:= atmost(3){i} c[i];      -- are at most 3 non-zero? (FALSE=0)
     atm  := atmost(2){i} (c[i]=0); -- are at most 2 equal to zero? (
        TRUE=1)
     exa  := exactly(4){i} c[i];    -- are exactly 4 non-zero? (TRUE=1)
     exa1:= exactly(3){i} c[i];     -- are exactly 3 non-zero? (FALSE=0)
  Write('results␣of␣all␣index-operators\n');
  Write('%7.2f␣%7.2f␣%7.2f␣%7.2f␣%7.2f␣%7.2f␣\n␣%d␣%d␣%d␣%d␣%d␣%d␣%d␣%d
      ␣␣\n',
    su,pr,ma,mi,pma,pmi,ex,fo1,fo2,atl3,atl4,atm,exa,exa1);
 end
```

30 Logical Operators (learn02a)

–– Run LPL Code , HTML Document –

Problem: This model shows further operators and functions of LPL.

Listing 30: The Complete Model implemented in LPL [2]

```
model learn02a "Logical Operators";
  set t := [1..4];
  parameter x{t} := [1 1 0 0];  y{t} := [1 0 1 0];
  Write('\n
␣␣␣␣␣--------------LOGICAL␣operators------------------------
␣␣␣␣␣x␣␣␣␣y␣␣␣|␣␣␣␣and␣␣or␣␣␣NAND␣␣nor␣␣XOR␣␣␣->␣␣␣␣<-␣␣␣<->
␣␣␣␣␣---------|---------------------------------------------\n');
  Write{t}(
  '␣␣␣␣␣%1d␣␣␣␣%1d␣␣␣|␣␣␣␣␣%1d␣␣␣␣%1d␣␣␣␣␣%1d␣␣␣␣%1d␣␣␣␣%1d␣␣␣␣␣%1d␣␣␣␣
     ␣%1d␣␣␣␣%1d\n',
     x,y,x and y,x or y,~(x and y),~(x or y),x xor y,x->y,x<- y,x<->y);
  --- Several examples of logical index operators
  set q := [1..16];
  binary parameter
     z{t,q} := [1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0
                1 1 1 1 0 0 0 0 1 1 1 1 0 0 0 0
                1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0
                1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0];
    b1{q} := and{t} z  "same as forall{} ...";
    b2{q} := nand{t} z "same as ~and{}";
    b3{q} := or{t} z   "same as exist{} ...";
    b4{q} := nor{t} z  "same as ~or{}";
    b5{q} := xor{t} z  "same as exactly(1){}";
    b6{q} := z[4,q] or xor{t} z;
    b7{q} := atleast(1) {t} z  "same as or{}";
    b8{q} := atmost (2) {t} z;
    b9{q} := exactly(3) {t} z;
  Write('\n\n');
  Write{q}('␣␣%2s␣␣%d␣%d␣%d␣%d␣%d␣%d␣%d␣%d␣%d␣\n', q,b1,b2,b3,b4,b5,b6,
     b7,b8,b9);
end
```

31 The within and in Operators (learn03)

—– Run LPL Code , HTML Document –

Problem: The keywords **within** and **in** are similar used in different contexts. (within was introduced in version LPL6.66 to differenciate two meanings.)

The first part of the model shows the **within** in an expression: `i within j` returns the position of element i within set j, zero if not found (note that the first position is 1).

The second part shows it in a indexed part as `{...}`. For example, `{i in s,...` introduces a *local* identifier `i` as a place-holder for `s` in the following part of the expression.

Listing 31: The Complete Model implemented in LPL [2]

```
model learn03 "The within and in Operators";
  ---- first example with the within-operator ----
  set i := [1..12];
  set j := [2..7];
  parameter x{i}:=i;
    r1{i | i within j}     := x;
    r2{i | ~(i within j)} := x;
    r3{j}  := x[j within i];
    r4{i}:= x[i within j];
  Writep(x,r1,r2,r3,r4);
  ---- second example with in-operator ----
  set s := [s1 s2 s3 s4 s5 s6 s7 s8 s9 s10 s11];
      t := [s2 s3 s4 s5 s12 ];
  parameter a{i in s,j in s | i<7 and (j=i+1 or j=i+2 or j=i+5)}:=1 "
       Compares position";
    b{s,t | t within s} :=1  "Compares the element names";
  Writep(a,b);
end
```

32 Some String Compare Operations (learn04)

–– Run LPL Code , HTML Document –

Problem: String comparision : string can be used as litterals within ' ' or as identifiers declared as string parameter. They can be used anywhere in an expression, where a number or another identifier is allowed. In general they return a zero if evaluated in an expression except in the following cases:

1. used in a applied index-list in place of an element. In this case it is tested if this element exists. If not, zero is returned. example: ... + x['apple','period4'] + ...
2. together with one of the six relational operators. (`<, <=, =, >, >=, <>`) example : `x < 'Period5'` (`x` must be of type STRING or set)

Listing 32: The Complete Model implemented in LPL [2]

```
model learn04 "Some String Compare Operations";
set
  i := [ KA EI FE KH ET FS ];
parameter
  a{i} := [ 1 2 3 4 5 6 ];
  b    := a['EI']  "reference element by name";
  b1   := a[2]     "same but reference by position";
  c    := a['KA'];
  c1   := a[1];
  d0   := '100' < '15'  "true";
  d1   := 100 < 15      "false";
  e{i} := i >  'FS'  "string comparison!";
  f{i} := i >= 'FS';
  g{i} := i <  'FS';
  h{i} := i <= 'FS';
  j{i} := i <> 'FS';
  k{i} := i =  'FS';
string parameter
  s    := ['Fette'];
  t{i} := [ 'Kalorien' , 'Eiweiss' , 'Fett' , 'Kohlenhydrate' ,
            'tierische␣Eiweisse' , 'Fett␣sichtbar' ];
parameter
  m{i} := t <= t[3];
  m1{i}:= t < t['FS'];
  n{i} := i < t;
  o{i} := s < t;
Writep(m); -- check others
end
```

33 Index options, and wrap around function (learn05)

—– Run LPL Code , HTML Document –

Problem: Note p and t are two basic sets that have nothing in common, although one can test whether an element from one is also in the other with the **within** operator. On the other hand, t1 is a subset (a relation) of p. Normally, it does not make sense to mix two basic sets. See what happens here by generating the EQU file (locally).

Listing 33: The Complete Model implemented in LPL [2]

```
model learn05 "Index options, and wrap around function";
set
  p :=      [0..14] ;
  t :=      [1..13] ;
  t1{p} := p<>'0' and p<>'14';
variable X{p};
constraint
  x0: sum{p} X;             --- omits index list
  x1: sum{p} X[p];          --- or: explicit index list
  x3: sum{i in p} X[i];     --- same as previous (using a local)
  y0: sum{t} X[t within p];  --- index replacement
  y1: sum{t1} X[t1];         --- same as previous
  y2: sum{i in t} X[i];      --- rather strange, but it works
  w0: sum{i in p} X[i-1];   --- no error for the first element!
  w1: sum{t1} X[t1-1];      --- that's interesting, isn't it?
  w2: sum{i in t1[i]} X[i-1];  --- that's the same
  v0: sum{i in p} X[i%#i+1];          --- +1 wrap around
  v1: sum{i in p} X[(i+#i-3)%#i+1];   --- -2 wrap around
solve;
--- another small example
   set i := [1..5];  k{i} := [1 3 5];
   parameter a{k} := [10 30 50];
   parameter b{i} := k;
   parameter c{i} := a;
   Writep(i,k,a,b,c);
end
```

34 Sort function (learn06)

—– Run LPL Code , HTML Document –

Problem: This example shows how to sort a vector a_i. The vector a is not sorted itself. The sort generates a permutation vector b of the same length as a and the sorted vector a can then be generated by the syntax $a[b]$ (or more explicit $a[b[i]]$.

Listing 34: The Complete Model implemented in LPL [2]

```
model learn06 "Sort function";
  set i:=1..10;
  parameter a{i}:=Trunc(Rnd(1,100));
  integer   b{i};
  Sort(a,b);
  Writep(a,b);
  Write{i}('%3d␣\t␣%5.2f␣\n',b,a[b]);
  set j:=1..5;
  parameter c{i,j}:=Trunc(Rnd(1,100));
            d{i,j};
  Sort(c,d);
  Write('\n');
  Write{i,j}('%3d␣\t␣%5.2f␣\n',d,c[d]);
end
```

35 Date/Time Type (learn07)

—- Run LPL Code , HTML Document –

Problem: Date parameters are defined as given in the model (see `datatime`). Note that internally a date is treated as a double and can be treated as such.

Output a date is done using the `%t...` formatting. One can also use the `Format()` function for generating a string of the date.

Listing 35: The Complete Model implemented in LPL [2]

```
model learn07 "Date/Time Type";
  date parameter datetime:=@2018-12-11T13:10:15;
  Write('A␣date␣and␣time:␣%tc␣,␣%tr\n', datetime,datetime);
  string parameter dt:=Format('%tc␣%tT', datetime,datetime);
  Write('dt=%s\n\n',dt);
  date parameter birthday := @1961-04-14;
  Write('Your␣birthday␣is␣:␣%tF\n', birthday);
  Write('You␣already␣live␣%d␣days.\n\n', Now()-birthday);
  set i:=1..30;
  Write{i}('At:␣%tF␣is␣your␣%5d-th␣birthday\n',
            birthday+1000*i,1000*i);
end
```

36 Documenting Models (learn08)

—– Run LPL Code , HTML Document –

Problem: This is a model with inline documentation text. The model documentation can be generated automatically using a compiler switch (`T` and `t` (see reference manual.

This is the documentation for the main model. The model is a simple linear program with 10 variables. All data are declared and defined inline, that is, they are defined within the model code.

Modeling Steps

Note that the code contains comments for each entity. In *lplw.exe*, press key F5 to generate the documentation. Make sure that the batch file *modeldoc.bat* contains the correct paths. .....

This is the LPL code:

Listing 36: The Complete Model implemented in LPL [2]

```
model learn08 "Documenting Models";
  /**
    * First a set @i@ is defined. It has 10 elements.
    * It is used to define the 10 variables.
    */
  set  i := [ 1..10 ]   "A set with 10 elements";
  /**
    * A variable vector @x@ defined the model variable.
    * it is indexed over @i@ (as $x_i$).
    */
  variable  x{i} "The number of different type of robots";
  /**
    * The following parameters define model data
    */
  parameter HC{i} := [ 5 5 4 5 6 5 7 8 4 7 ] "Component time";
      HM{i} := [ 4 8 5 6 4 8 7 6 5 3 ] "Mounting time";
      HT{i} := [ 6 2 4 6 3 4 5 2 5 3 ] "Testing time";
      Ordered{i} := [ 20 15 7 6 5 8 9 8 7 5 ] "Quantity ordered";
      Price{i}  := [ 300 200 100 50 50 100 200 100 400 200 ];
  /**
    * Next, four model constraints are defined, three of them are
    * capacity restrictions, and the last one is a demand requirement.
    */
  constraint
    Component :sum{i} HC[i] * x[i] <= 3500 "Component building";
    Mounting:  sum{i} HM[i] * x[i] <= 4800 "Mounting robots";
    Testing:   sum{i} HT[i] * x[i] <= 3000 "Testing robots";
    Order{i}:  x[i]                >= Ordered[i] "Ordered";
  /**
    * Finally, we define the objective function, which is to
    * maximize the revenue (or total sell).
    */
  maximize revenue: sum{i} Price[i]*x[i] "Maximize the profit";
  /**
    * Data and the result are finally written to a file,
    * the so called NOM-file. It has the same name as the
    * model file (here @doc.lpl@) with extension 'nom'.
    * Hence, the results are written to the file @doc.nom@.
    */
  Writep(revenue,x,HC,HM,HT);
  /** We also can define submodels within main models. The next
```

```
    * declaration is such a submodel, called @submodel@. However
    * It is not used. It is only for documentation purpose.
    */
  model submodel "a submodel";
    /** one may declare other entities here... */
    set i;
  end
end
```

And this is what is generated :

The Model

function $learn08$

First a set i is defined. It has 10 elements. It is used to define the 10 variables.

set i "A set with 10 elements"

A variable vector x defined the model variable. it is indexed over i (as x_i).

var x_i "The number of different type of robots"

The following parameters define model data

param HC_i "Component time"

param HM_i "Mounting time"

param HT_i "Testing time"

param $Ordered_i$ "Quantity ordered"

param $Price_i$

Next, four model constraints are defined, three of them are capacity restrictions, and the last one is a demand requirement.

s.t. $Component$: $\sum_i HC_i \cdot x_i \leq 3500$ "Component building"

s.t. $Mounting$: $\sum_i HM_i \cdot x_i \leq 4800$ "Mounting robots"

s.t. $Testing$: $\sum_i HT_i \cdot x_i \leq 3000$ "Testing robots"

s.t. $Order_i$: $x_i \geq Ordered_i$ "Ordered"

Finally, we define the objective function, which is to maximize the revenue (or total sell).

maximize $revenue := \sum_i Price_i \cdot x_i$ "Maximize the profit"

Data and the result are finally written to a file, the so called NOM-file. It has the same name as the model file (here `doc.lpl`) with extension 'nom'. Hence, the results are written to the file `doc.nom`.

Writep$(revenue, x, HC, HM, HT)$

We also can define submodels within main models. The next declaration is such a submodel, called `submodel`. However It is not used. It is only for documentation purpose.

```
        **function** *submodel*

                one may declare other entities here...

            **set** *i*

        **end**

    **end**
```

37 Call Submodel within Solver (Gurobi) (learn10)

—– Run LPL Code , HTML Document –

Problem: This model shows the effect of the `CALLBACK` option for the solver (Gurobi).

(Note that the correct Gurobi version must be installed!).

First it defines the solver interface string `gurobiLSol` (see manual Chap 9.2) This is necessary for the very last parameter (5000), which is the SIP22 – meaning 20secs. The solver is called with the option `'CALLBACK=callit'` which means that the solver must call the submodel `callit` on a regular base. The first time the solver calls it is when it gets its first feassible solution. Then it calls it each time when a new integer feasible solution is found but not before 20secs have elapsed.
This options allows the modeller to save intermediary solutions for a lengthy calculation of a very hard IP problem. Try out different value for the SIP22. The default is 0secs, which means that the submodel is called inside the solver each time when a new integer feasible solution has been discovered.
In our case, the `callit` just adds a line to the LOG-file when it was called.

Listing 37: The Complete Model implemented in LPL [2]

```
model learn10 "Call Submodel within Solver (Gurobi)";
  string parameter gurobiLSol1 := ',,lib:C:/gurobi900/win64/bin/
      gurobi90,gurobi.prm,,TimeLimit=100;OutputFlag=0,,,,,,,,,,,LP;MIP
      ;QP;iQP;QCP;iQCP,,,,5000';
  SetSolver(gurobiLSol1,'CALLBACK=callit');
  set i,j:=1..16;
      s:=1..15;
  parameter c{i,j,s}:=Trunc(Rnd(1,30));
  binary variable x{i,j,s|i<>j};
  constraint
    A{i,j|i<j}: sum{s} (x[i,j,s]+x[j,i,s]) = 1;
    B{i,s}: sum{j} (x[i,j,s]+x[j,i,s]) = 1;
  minimize obj: sum{i,j,s} c*x;
  model callit;
    parameter n;;
    n:=n+1;
    Write('obj␣=␣%2d␣after␣%d␣secs\n', sum{i,j,s} c*x,GetParam(1)/1000
        );
  end
end
```

38 Sparsity Check (learn11)

—- Run LPL Code , HTML Document –

Problem: Define a very sparse set $S_{i,j,k,p}$ indexed over four indices of size 100. So full cadinality of S is 10^8. However, the cardinality of sparse S is only about 500000 elements.

Constraint `A` and `B` below are completely identical. Note that the indexes are not in the same order as defined in `s{i,j,k,p}`. LPL needs 60secs to generate constraint `A`, but only 5secs for constraint `B`.

It is therefore important for large models how to exploit sparsity.

Listing 38: The Complete Model implemented in LPL [2]

```
model learn11 "Sparsity Check";
  set
    i:=1..100;
    j:=1..100;
    k:=1..100;
    p:=1..100;
    S{i,j,k,p}:=if(Rnd(0,1)<=0.005,1);
  variable  x{i,j,k,p};
  constraint A{j,p}: sum{i,k|S} x = 2;
  constraint B{j,p}: sum{(i,k) in S} x = 2;
  solve;
end
```

39 Reading Relations (learn12)

—- Run LPL Code , HTML Document –

Problem: How to read “relations” from text files.

Listing 39: The Complete Model implemented in LPL [2]

```
model learn12 "Reading Relations";
  --define three basic sets
  set  i; j; k;
  --define four relations
  set ij{i,j}; ik{i,k}; jk{j,k}; ijk{i,j,k};
  parameter a{i,j,k};
  -- reading them from file
  Read{i,j,k}('learn12.txt', i,j,k,ij,ik,jk,ijk,a);
  Writep(i,j,k,ij,ik,jk,ijk,a);
end
```

Note: reading relations does *not* physical read in text files. So in fact the Read instruction only reads i, j, and k, (and a). The 4 relations are derived. This is a powerful option to generate sparse relations. The content of the text file ’learn12.lpl’ is as follows:

```
  // a data set (i,j,k,a) for learn12.lpl
  1 a A  12
  1 b A  13
  2 b B  15
  3 a A  17
  3 c C  19
  4 a A  21
  4 b A  11
  4 b B   8
  5 a B   7
  5 b A  -3
  5 c C  33
```

40 A Small Data Cube I (learn13)

—– Run LPL Code , HTML Document –

Problem: Open this model with lplw.exe an play around with the 4-dimensional pivot tables.

Listing 40: The Complete Model implemented in LPL [2]

```
model learn13 "A Small Data Cube I";
  set i := [ 1 2 ];
      j := [ a b c ];
      k := [ w x y z ];
      h := [ p q r s t];
  parameter a{i,j,k,h} := if(Rnd(0,1)<0.3,Trunc(Rnd(1,9)));
/*    [ 3 . 5 . . 9 ,
      4 5 6 . . . . ,
      2 1 7 3 3 . ,
      . 8 . . 4 5 ];*/
  b{i,j,k} :=  a[i,j,k,'p'];   -- select / project ;
  c{i,k,h} :=  sum{j} a;
  d{i,j}   :=  sum{k}(max{h} a);
  e{k,h}   :=  a['1','a',k,h];
  f        :=  sum{i,j,k,h} a;
  g{i,j,k,h | a>4} := a;
  x{i,k,h}  := sum{j | a<5} a;
  set ijkh,I{i,j,k,h} := a;
      ijKh{i,j,k,h} := if (i=1 and j=2 and h=2,1);
  variable v{I[i,j,k,h]} := b[i,j,k]*c[i,k,h];
    z{ijKh};
  constraint C{i,j,k,h}: 2*v + z[i,j,k,h] =10;
  minimize obj: sum{ijKh} z;
  --write a;
end
```

41 A Small Data Cube II (learn13a)

–– Run LPL Code , HTML Document –

Problem: Open this model with lplw.exe an play around with the 4-dimensional pivot table.

Listing 41: The Complete Model implemented in LPL [2]

```
model learn13a "A Small Data Cube II";
  set product, p := [1..10];
    location, i,j:= [A B C D E F G];
  parameter transportationCost{p,i,j|i<>j}
     := if(Rnd(0,1)<0.15 , Rnd(0,60));
  ----
  set share, s := [s1 s2 s3 s4 s5];
    period, t := [t1 t2 t3 t4 t5 t6];
  parameter returnValue{s,t} := Rnd(0,1);
  ----
  set machine, m := [m1 m2 m3 m4];
    mode, o := [o1 o2 o3];
  parameter hours{m,p,o,t}
     :=  Trunc(if(Rnd(0,1)<0.35 , Rnd(0,10)));
  Writep(hours,o);
end
```

42 Multiple bounds of variables (learn14)

—- Run LPL Code , HTML Document –

Problem: Bounds can be assigned by the bound attribute `[a..b]` at the declaration of an entity. Bounds on varibales can also be assigned by constraints. Generate the EQU-file to see the result.

Listing 42: The Complete Model implemented in LPL [2]

```
model learn14 "Multiple bounds of variables";
  set i := [1..10];
  parameter a{i} := i;
     b{i} := 10+i;
  variable
    x [1..99]        "x has lower/upper bound of 1 and 99";
    y{i} [a..b]      "The lower/upper bound of y is given by the vectors
        a and b";
    z{i} [i..10+i]   "Any expression can be used as a bound";
  constraint
    r1: x <= 100    "This constraint will have no effect, upper bound is
        still 99";
    r2: x >= 2      "This overrides the lower bound of 1";
    r3{i}: y <= 15 "Some upper bounds of y are reassigned";
    r4{i}: z >= 4  "Some lower bounds of z are reassigned";
  maximize obj1: x + sum{i}(y+z) "The solution is at the upper bounds";
  Writep(x,y,z);
  minimize obj2: x + sum{i}(y+z) "The solution is at the lower bounds";
  Writep(x,y,z);
end
```

43 Submodels and Encapsulation (learn15)

–— Run LPL Code , HTML Document –

Problem: This model declares four (sub)models (m1-m4). The executable part of the main model runs the four models one after the other. Note that the same idetifier can be redeclared in different models.

Listing 43: The Complete Model implemented in LPL [2]

```
model learn15 "Submodels and Encapsulation";
  m1; m2; m3; m4; --execute the four models defined below
  Write('----writes␣from␣the␣main␣model\n');
  Writep(m1.a,m4.a);
  model m1 "first model";
    set
      i := [i1 i2 i3 i4 i5];
      j := [j1 j2 j3 j4 j5];
      m{i,j} := [ i1 j1 , i1 j4 , i1 j5 , i2 j2 , i3 j2 , i4 j5 ];
    parameter
      d{i,j} := 1;
      a{m} := m;                  -- same as : a{i,j | m[i,j]} := m;
      b{m[i,j] | i<j} := i*j;     -- same as : b{i,j | m[i,j] and i<j} :=
          i*j;
      c{m,i | i<4} := i;          -- same as : c{k in i,j,i | m[k,j] and i
          <4} := i;
    Write('writes␣from␣the␣model␣m1\n');
    Writep(i,j,m,d,a,b,c);
  end
  model m2 "second model";
    set
      i :=     [ A1 A2 A3 A4 ];
      j :=     [ B1 B2 B3 ];
      k{i} :=  [ A2 A4 ];
      m{i,j} :=[ A2 B2 , A2 B3 , A4 B2 ];
    parameter
      a{m} := [1 2 3];
      b{m,k} := / A2 B3 A4 2 , A4 B2 A2 3 , A2 B2 A2 5 /;
      c{m} := / A2 B3 1 , A1 B3 4 /;
        /* note that the tuple (A1 B3) is not in (m) , therefore it
           will
           never show up */
      d{m} := 4;
    Write('writes␣from␣the␣model␣m2\n');
    Writep(i,j,k,m,a,b,c,d);
  end
  model m3 "third model";
    set
      i := [ i1 i2 i3 i4 i5 ];
      j := [ j1 j2 j3 ];
      k := [ k1 k2 k3 k4 ];
      m{i} := [ i1 i2 i4 ];
      p{j} := [ j2 j3 ];
      n{i,j} := [ i1 j2 , i2 j2 , i4 j3 ];
      o{i,j,j} := [ i1 j2 j2 , i2 j2 j3 , i4 j3 j3 ];
    parameter
      a{k,o,n} := k*o*n ;
      b{o,o,i} := o*i ;
    Write('writes␣from␣the␣model␣m3\n');
    Writep(i,j,k,m,p,n,o,a,b);
  end
  model m4 "fourth model";
    set
      i := [ A1 A2 A3 A4 A5 A6 A7 A8 A9 ];  -- a basic set
      j{i} := [ A2 A3 A4 A5 A6 A7 A8 A9 ];  -- a derived subset of set
          i
      k{i} := [ A3 A4 A5 A6 A7 A8 ];        -- a derived subset of set
          i
      p{i} := [ A4 A5 A6 ];                 -- a derived subset of set
          i
      m :=  [ A3 A4 A5 A6 A7 A8 ];          -- m is an independent
          basic set
            -- note: m has nothing in common with any of the above sets
                !
    parameter
      simple_i{i} := i;  -- there is nothing special about these
          expressions
      simple_j{j} := j;
      simple_k{k} := k;
      simple_p{p} := p;
       /* the complication enters only, if the sets are mixed.
          Consider the following expressions */
      a{i} := 1000*i;
      b{i} := 1000*i + 100*j;
      c{i} := 1000*i + 100*j + 10*k;
      d{i} := 1000*i + 100*j + 10*k + p;
       /* you should read them as
          a{i} := 1000*i;
          b{i} := 1000*i + 100*j[i];   -- if i=1 then j[i] is zero.
          c{i} := 1000*i + 100*j[i] + 10*k[i];
          d{i} := 1000*i + 100*j[i] + 10*k[i] + p[i];
            which is also legal syntax in LPL. */
      e{i} := i within m;   -- the in operator
      f{m} := m within i;
     /* Note: "e{i} := m" (or f{m}=i) would produce an error, since m
        cannot
        by bound to i because they are two independent basic sets.
        But "m in i" returns the position of m within i; it is 0, if a
        corresponding m is not in i. */
    Write('writes␣from␣the␣model␣m4\n');
    Writep(i,j,k,p,m);
    Writep(simple_i,simple_j,simple_k,simple_p,a,b,c,d,e,f);
  end
end
```

44 Again, some non-trivial relations (learn16)

–– Run LPL Code , HTML Document –

Problem: A example with relations

Listing 44: The Complete Model implemented in LPL [2]

```
model learn16 "Again, some non-trivial relations";
  --- a first example: compare 'ge0' and 'ge1' ----
  set
    i := [1..10];
    j := [1..10];
    k := [1..2];
    s{i,j} := [ 5 5, 5 7, 5 9 ] "a 2-dimen. sparse relation";
  string parameter
    typ{k,s} := [ '<' '=>' '<' '=' '<' '>' ] "a 3-dimen table,note:
        cardinality is 6";
  parameter
    ge0{k,i,j | typ[k,s[i,j]]='<'} := 100*k+10*i+j;
    ge1{k,s[i,j] | typ[k,s]='<'} := 100*k+10*i+j;
  /* both, ge0 and ge1, return the same values. But there is a subtile
     difference: ge0 has a cardinality of 200, ge1 has a cardinality
     of only 6. Therefore, ge1 is more compact and much quicker to work
          through.
     Note also, that i and j in ge1 are locals, the globals i and j
     are not 'visible'. In ge0, however, i and j are globals. */
  ---- another example ----
  set
    ii,p,q  := [I1 I2 I3 I4 I5 I6 I7 I8 I9 I10];
    jj     := [J1 J2 J3 J4 J5 J6];
    kk{ii} := [ I5 I6 I7 ];
    mm{jj} := [ J2 J4 J6 ];
  parameter
    a1{p,q|kk[p] and mm[q]} := 1;
    a2{kk,mm} := 1;
  Write('(This␣model␣outputs␣nothing,␣run␣it␣locally\n');
end
```

45 Expressions and Constraints (learn17)

—- Run LPL Code , HTML Document –

Problem: Active constraints are model constraints that are passed to the solver; inactive entities are not passed to the solver. A way to select active constraints is through subject to: A list of active constraints is given by (id,id,...) A list of inactive constraints is given by (id, id, id,...).

Part of a constraint can be declared in 'expression'.

Listing 45: The Complete Model implemented in LPL [2]

```
model learn17 "Expressions and Constraints";
  variable X; Y; Z;
  expression
    B:= 10;
    C:= 11;
    A:= B+C;
    D:= X+Y;
  constraint
    E : D - Z = 3;
    F : D + D + Z = A;
    G  : 3*D + 2*Z >= A+10;
  minimize obj1: X-Y subject_to E,F;
  Write('OBJ1=%3d␣,␣X=%4.1f␣,␣Y=%4.1f␣,␣Z=%4.1f\n', X-Y,X,Y,Z);
  minimize obj2: X-Y  subject_to  learn17,~G;    --same as before
  Write('OBJ2=%3d␣,␣X=%4.1f␣,␣Y=%4.1f␣,␣Z=%4.1f\n', X-Y,X,Y,Z);
  minimize obj3: X-2*Y  subject_to  G,F;
  Write('OBJ3=%3d␣,␣X=%4.1f␣,␣Y=%4.1f␣,␣Z=%4.1f\n', X-2*Y,X,Y,Z);
end
```

46 GetValue Function (learn20)

–– Run LPL Code , HTML Document –

Problem: The function `GetValue()` returns various values of a entity. If `x` is a variable then `GetValue(x,3)` returns the dual value of `x` (if available).

Listing 46: The Complete Model implemented in LPL [2]

```
model learn20 "GetValue Function";
  set i := [1..10];  j := [1..9];
  parameter A{i,j} := if(Rnd(0,1)<0.25, Rnd(0,5));
            c{j}   := Trunc(Rnd(10,100));
            b{i}   := Trunc(if(Rnd(0,1)<0.75, Rnd(0,20)));
  variable  x{j} [1..50];
  constraint R{i} : sum{j} A*x >= b;
  minimize obj    : sum{j} c*x;
  Write('j␣␣␣␣␣␣␣␣␣␣␣␣␣␣=␣%6s\n',{j} j);
  Write('Value␣of␣x␣␣␣␣␣=␣%6.1f\n',{j} x);
  Write('GetValue(x,0)␣␣=␣%6.1f\n',{j} GetValue(x,0));
  Write('GetValue(x,1)␣␣=␣%6.1f\n',{j} GetValue(x,1));
  Write('GetValue(x,2)␣␣=␣%6.1f\n',{j} GetValue(x,2));
  Write('GetValue(x,3)␣␣=␣%6.1f\n',{j} GetValue(x,3));
  Write('GetValue(R,3)␣␣=␣%6.1f\n',{i} GetValue(R,3));
  Write('GetValue(R,4)␣␣=␣%6.1f\n',{i} GetValue(R,4));
  Write('GetValue(R,5)␣␣=␣%6.1f\n',{i} GetValue(R,5));
  Write('GetValue(R,6)␣␣=␣%6.1f\n',{i} GetValue(R,6));
  Write('GetValue(R,7)␣␣=␣%6.1f\n',{i} GetValue(R,7));
  Write('GetValue(R,8)␣␣=␣%6.1f\n',{i} GetValue(R,8));
end
```

47 create a SQL script, test sparcity (locally only) (learn21)

—- Run LPL Code , HTML Document –

Problem: Run this model locally with the interpreter option 'q'. This will generate two files: 'learn21.sql' and 'learn21.sq2'. The first is a complete executable SQL script that can be used to create a database. The second file is the corresponding read/write listing in LPL syntax able to read write to/from that database.

Listing 47: The Complete Model implemented in LPL [2]

```
model learn21 "create a SQL script, test sparcity (locally only)";
  -- create the SQL-script and compare it to the LPL model
  set i; j;
      ij{i,j} := [ i1 j1, i2 j2, i3 j3, i3 j4];
  parameter
    aij{i,j} := / i1 j2 1 , i2 j3 2 , i3 j4 3 , i3 j1 4, i2 j1 5/;
    bij{i,j} := / i1 j1 10, i2 j3 20, i2 j4 30, i3 j2 40/;
end
```

48 GetAttr Function (two parameters) (learn22)

—– Run LPL Code , HTML Document –

Problem: Implements reflection! The function `GetAttr()` returns a statement attribute (see chap 4.1 in the reference manual. For example, if `X` is an integer variable then `GetAttr(X,2)` returns the string ’integer’.

Listing 48: The Complete Model implemented in LPL [2]

```
model learn22 "GetAttr Function (two parameters)";
  // GetAttr with TWO parameters
  set i := [i1 i2 i3 i4 i5 i6 i7 i8 i9 i10];
  parameter a{i}:=i*10;
  integer variable
       X,a1,a2{i|a>=50}
       default 20
       priority i+2
       [i..i^2]
       "comment"
       'a␣quote'
       frozen
       := 100+i^4;
  Write('GetAttr(a,3)␣␣=␣%s\n\n',GetAttr(a,3));
  Write('GetAttr(X,2)␣␣=␣%s\n\
␣␣␣␣␣␣␣␣␣GetAttr(X,3)␣␣=␣%s\n\
␣␣␣␣␣␣␣␣␣GetAttr(X,4)␣␣=␣%s\n\
␣␣␣␣␣␣␣␣␣GetAttr(X,5)␣␣=␣%s\n\
␣␣␣␣␣␣␣␣␣GetAttr(X,7)␣␣=␣%s\n\
␣␣␣␣␣␣␣␣␣GetAttr(X,8)␣␣=␣%s\n\
␣␣␣␣␣␣␣␣␣GetAttr(X,9)␣␣=␣%s\n\
␣␣␣␣␣␣␣␣␣GetAttr(X,10)␣=␣%s\n\
␣␣␣␣␣␣␣␣␣GetAttr(X,12)␣=␣%s\n\
␣␣␣␣␣␣␣␣␣GetAttr(X,13)␣=␣%s\n\
␣␣␣␣␣␣␣␣␣GetAttr(X,14)␣=␣%s\n\
␣␣␣␣␣␣␣␣␣GetAttr(X,15)␣=␣%s\n\
␣␣␣␣␣␣␣␣␣GetAttr(X,16)␣=␣%s\n\
␣␣␣␣␣␣␣␣␣GetAttr(X,18)␣=␣%s\n\
␣␣␣␣␣␣␣␣␣GetAttr(X,19)␣=␣%s\n\
␣␣␣␣␣␣␣␣␣GetAttr(X,20)␣=␣%s\n\
␣␣␣␣␣␣␣␣␣GetAttr(X,22)␣=␣%s\n\
␣␣␣␣␣␣␣␣␣GetAttr(X,23)␣=\n%s\n\
␣␣␣␣␣␣␣␣␣GetAttr(X,24)␣=\n%s\n\
␣␣␣␣␣␣␣␣␣GetAttr(X,25)␣=\n%s\n\
␣␣␣␣␣␣␣␣␣GetAttr(X,26)␣=\n%s\n',
    GetAttr(X,2),GetAttr(X,3),GetAttr(X,4),GetAttr(X,5),GetAttr(X,7),
    GetAttr(X,8),GetAttr(X,9),GetAttr(X,10),GetAttr(X,12),
    GetAttr(X,13),GetAttr(X,14),GetAttr(X,15),GetAttr(X,16),
    GetAttr(X,18),GetAttr(X,19),GetAttr(X,20),GetAttr(X,22),
    GetAttr(X,23),GetAttr(X,24),GetAttr(X,25),GetAttr(X,26));
end
```

49 GetAttr Function (one parameter) (learn22a)

—- Run LPL Code , HTML Document –

Problem: The `GetAttr()` function can also be called with a single parameter. In this case it returns the statement attribute of the focused entity. (see learn22[18].

Listing 49: The Complete Model implemented in LPL [2]

```
model learn22a "GetAttr Function (one parameter)";
  // GetAttr with ONE parameters
  set i := [i1 i2 i3 i4 i5 i6 i7 i8 i9 i10];
  parameter a{i}:=i*10;
  integer variable
       X,a1,a2{i|a>=50}
       default 20
       priority i+2
       [i..i^2]
       "comment"
       'a␣quote'
       frozen
       := 100+i^4;
  SetFocus(a);
  Write('GetAttr(3)␣(focus␣is␣a)␣=␣%s\n\n',GetAttr(3));
  SetFocus(X);
  Write('GetAttr(2)␣␣=␣%s\n\
␣␣␣␣␣␣␣␣␣GetAttr(3)␣␣=␣%s\n',
    GetAttr(2),GetAttr(3));
end
```

[18]https://lpl.matmod.ch/lpl/Solver.jsp?name=/learn22

50 GetName Function (learn23)

—– Run LPL Code , HTML Document –

Problem: The `GetName` function returns the name of an instanced entity in various formats. Example: if `X` is defined as in the model below then `GetName(X[1,2],0)` returns `'X[i1,B]'` and so on. Run the model and check.

Listing 50: The Complete Model implemented in LPL [2]

```
model learn23 "GetName Function";
  set i := [i1 i2];
      j := [A, B, C];
  string parameter I{i}:=['firstI', 'secondI'];
                   J{j}:=['1stJ', '2ndJ', '3rdJ'];
  parameter II{i} := [100 200];
  StringToSet(i,I);  -- link i to I
  --StringToSet(i,II);  -- link i to II //try this
  StringToSet(j,J);  -- link j to I
  variable X{i,j}:=1;
  Write{i,j}('GetName(X,0)␣␣=␣%s\n',GetName(X,0));
  Write{i,j}('GetName(X,1)␣␣=␣%s\n',GetName(X,1));
  Write{i,j}('GetName(X,2)␣␣=␣%s\n',GetName(X,2));
  Write{i,j}('GetName(X,3)␣␣=␣%s\n',GetName(X,3));
  Write{i,j}('GetName(X,4)␣␣=␣%s\n',GetName(X,4));
  Write{i,j}('GetName(X,5)␣␣=␣%s\n',GetName(X,5));
  Write{i}('GetName(II,4)␣␣=␣%s\n',GetName(II,4));
  Write{i}('GetName(II,5)␣␣=␣%s\n',GetName(II,5));
  Write{i}('GetName(II,6)␣␣=␣%s\n',GetName(II,6));
end
```

51 GetParams Function (learn24)

—- Run LPL Code , HTML Document –

Problem: The function `GetParams()` returns a ”global” parameter of the model. For example, `GetParams(8)` returns the actual LPL version you are running, etc.

Listing 51: The Complete Model implemented in LPL [2]

```
model learn24 "GetParams Function";
  Write('GetParams(0)␣␣␣=␣%s\n',GetParamS(0));
  Write('GetParams(1)␣␣␣=␣%s\n',GetParamS(1));
  Write('GetParams(2)␣␣␣=␣%s\n',GetParamS(2));
  Write('GetParams(3)␣␣␣=␣%s\n',GetParamS(3));
  Write('GetParams(4)␣␣␣=␣%s\n',GetParamS(4));
  Write('GetParams(5)␣␣␣=␣%s\n',GetParamS(5));
  Write('GetParams(6)␣␣␣=␣%s\n',GetParamS(6));
  Write('GetParams(7)␣␣␣=␣%s\n',GetParamS(7));
  Write('GetParams(8)␣␣␣=␣%s\n',GetParamS(8));
  Write('GetParams(9)␣␣␣=␣%s\n',GetParamS(9));
  Write('GetParams(10)␣␣=␣%s\n',GetParamS(10));
  Write('GetParams(11)␣␣=␣%s\n',GetParamS(11));
  Write('GetParams(12)␣␣=␣%s\n',GetParamS(12));
  Write('GetParams(13)␣␣=␣%s\n',GetParamS(13));
  Write('GetParams(15)␣␣=␣%s\n',GetParamS(15));
  Write('GetParams(16)␣␣=␣%s\n',GetParamS(16));
  Write('GetParams(17)␣␣=␣%s\n',GetParamS(17));
end
```

52 Split Function (learn25)

–– Run LPL Code , HTML Document –

Problem: The function `Split` splits (or tokenizes) a string into several parts separates by a character (the splitting character). For example, if the string is `'aa,bb,ccc,d'` and the splitting parameter is `,` (a comma), then the function can tokenize it into `'aa'`, `'bb'`, `'ccc'`, and `'d'`. The result must be a parameter. Example:

```
string a; b; c; d;
Split('aa,bb,ccc,d',',',a,b,c,d);
```

The string parameters `a`, `b`, `c`, and `d` receive the parts as string. If the parts are numbers (or dates) then a type cast is automatically done. Example:

```
integer a; b; c; d;
Split('12,1234,1,45',',',a,b,c,d);
```

The parameters `a`,`b`,`c`,and `d` will have the numerical values 12, 1234, 1, and 45.

Another form of the Slip function to tokenize a (list of) strings into a single string table. Example:

```
string parameter S{i} := ['a,b,c,d,e,f' '1,2,3' 'AA,BB,CC,DD'];
set k := [1..7];
string parameter A{i,k};
{i}Split(S,',',{k} A);
```

In this case the string parameter `A{i,k}` will receive all string parts spitted. Run the model to see the effects. Such a split may be especially interesting in a Read statement.

Listing 52: The Complete Model implemented in LPL [2]

```
model learn25 "Split Function";
  set i:=[1..3];
  string parameter s{i} := ['a,b,c,d' '1,2,3,4' 'AA,BB,CC,DD'];
  string parameter a{i}; b{i}; c{i}; d{i};
  {i}Split(s,',',a,b,c,d);
  Writep(a,b,c,d);
  string parameter S{i} := ['a,b,c,d,e,f' '1,2,3' 'AA,BB,CC,DD'];
  set k := [1..7];
  string parameter A{i,k};
  {i}Split(S,',',{k} A);
end
```

53 String Functions (learn26)

–– Run LPL Code , HTML Document –

Problem: Note there is no type cast function that transforms a number into a string. To transform a number into a string use the concatenation operator & with an empty string, such as `''&12.3` which returns a string `'12.3'`.

Listing 53: The Complete Model implemented in LPL [2]

```
model learn26 "String Functions";
  parameter d := Strdate('2018-12-20');
  parameter d1:= Strdate('1899-12-31');
  parameter r := Strfloat('56.17');
  parameter t := Strfloat('1,2.3,4.5,5',',',3);
  parameter a := Strlen('abcd');
  parameter b := Strpos('bc','abcdef');
  string parameter c := Strreplace('abcedf','bc','-ccbbcbc-');
  string parameter s := Strsub('abcdefg',2,3);
  Writep(d1,r,t,a,b,c,s);
  Write('%s\n','Length␣of\'abcd\'␣is␣' & a);
  Write('%tc\n', d);
end
```

To transform a string into a number type, use the function `Strfloat`. This function can also extract the n-th occurence of a substring. For example the parameter `t` in the model was extracted from the 3-th substring, separarted by the comma charcater (,).

54 String Operations and Format (learn26a)

–– Run LPL Code , HTML Document –

Problem: Show some string operations and the Format function

Listing 54: The Complete Model implemented in LPL [2]

```
model learn26a "String Operations and Format";
  set i:=[1..5]; q:=[1..4];
  set IQ{i,q}:= if(Rnd(0,1)<=.5,1,0);
  string parameter x{i,q} := Format('i%1sq%1s', i, q);
  string parameter concatB{q}:='';
  for{i} do concatB{q|IQ}:=concatB & x[i,q] & ';'; end
  string parameter concatB1{q}:=Format('%4s%1s', {i|IQ} x,';');
  Writep(concatB,concatB1);
  Write('\n');
  string parameter  a{i} := ['abcdef', '12345c789', 'qwcffc', 'ggggggh'
     , 'cuitrtzuiop'];
  integer parameter lenA{i} := Strlen(a);
  integer parameter posC{i} := Strpos('c',a);
  string parameter  subA{i} := Strsub(a,3,20);
  string parameter formA := Format('Total:␣%8.2f␣%6s␣%7d␣␣Date:␣%tc',
     234.567, 'Uiii', 456, Now());
  string parameter formB := Format{i}('I:%2s␣\t␣i^4=%3d␣␣\n', i,i^4);
  Writep(lenA,posC,subA);
  Write('\n/%s/\n\n', formA);
  Write(formB);
  Write('---The␣same␣again---\n');
  Write{i}('I:%2s␣\t␣i^4=%3d␣␣\n',i,i^4);
end
```

55 Functions NextFocus, NextPosition (learn27)

—– Run LPL Code , HTML Document –

Problem: This model show the use of the functions `NextFocus` and `NextPosition`. The first part of the model is a small production/distribution model containing three set and two variable declaration.

The goal is to output all sets and the corresponding elements. This is done in the second part: First the focus is set to the beginning of the model by calling `SetFocus()`. Then a loop is repeated as long as `NextFocus(0)` returns true (not zero). The function jumps to the next basic set (0 is the parameter value for genus 0 – which is the basic set. If there is no more basic set then the function `NextFocus(0)` returns false (0) and the loop ends. The inner loop jumps from one data entry to the next in lexicographical ordering. In this case, it traverses all elements of the focused set. The function `NextPosition` returns true as long as a next entry exists, otherwise it returns false (0). Note that we use *GetValueS()* to return the element namem, because it is a string.

The third part outputs all variables with their value and dual value. The mechanism is the same as in the second part. In this case, however the function `NextFocus(3)` jumps from a variable entity to the next (3 is the parameter for genus 3 – that is, the variable. Note that in this case we must use `GetValue()` to retrieve the values because they are numerical.

Listing 55: The Complete Model implemented in LPL [2]

```
model learn27 "Functions NextFocus, NextPosition";
  set s := [ SS  HD ];
      d := [ BA  PH  WA  RI ];
      p := [ P1  P2 P3 ];
   parameter trcost{s,d} := [
          3.52  9.47  0.38  8.63, 2.04  6.61  7.22  9.97 ];
    prcost{s,p} :=  [ 4.22  5.05  4.60 , 1.45  2.45  2.03 ];
    prCap{s}    :=  [ 5000  6000 ];
    orders{d}   :=  [ 120  100   30  234 ];
  variable PR{p,s}; SH{p,s,d};
  constraint
    Capacity{s} : sum{p}  PR <= prCap;
    Balance{p,s}: PR = sum{d} SH;
    Demand{p,d} : sum{s} SH >= orders;
  minimize costs: sum{p,s} (prcost*PR + sum{d} trcost*SH);
  Write('LIST␣OF␣ALL␣BASIC␣SETS\n');
  SetFocus();
  while NextFocus(0) do
    Write('%s␣%s␣(alias:␣%s)\n' , GetAttr(3),GetAttr(4),GetAttr(12));
    while NextPosition do  Write('␣␣%s\n', GetValueS(0)); end
  end
  Write('\nLIST␣OF␣ALL␣VARIABLES\n');
  SetFocus(0);
  while NextFocus(3) do
    Write('%s␣%s\n' , GetAttr(3) , GetAttr(4));
    while NextPosition do Write('%s␣=␣%5.2f␣␣(dual=%4.2f)\n',
                          GetName(0),GetValue(0),GetValue(3));
    end
  end
end
```

56 Multiple Snapshots (learn29)

–– Run LPL Code , HTML Document –

Problem: This model shows the use of multiple snapshots. A snapshot file is a compact file that stores all the data (variable values included) from a model into a file. It is therefore – at the moment of writing – a "snapshot" of the actual state. The instruction:

```
write to '<filename>.sps';
```

generates automaticcally such a file. All you need is, thta the file extension *must* be: `sps`.

Listing 56: The Complete Model implemented in LPL [2]

```
model learn29 "Multiple Snapshots";
  set i; j; parameter a{i,j};;
  test1;
  test2;
  test3;
  test4;
  model test1;
    i:=[A B D];    j:=[a b c d];
    a{i,j}:= [1 2 3 4 5 6 7 8 9 10 11 12];
    Write('1.sps');
  end
  model test2;
    i:=[A C B];    j:=[a b c d];
    a{i,j}:= [1 2 3 4 5 6 7 8 9 10 11 12];
    a{i,j}:=10*a;
    Write('2.sps');
  end
  model test3;
    i:=[D G A];    j:=[b c d a];
    a{i,j}:= [1 2 3 4 5 6 7 8 9 10 11 12];
    a{i,j}:=100*a;
    Write('3.sps');
  end
  model test4;
    Read('1.sps');
    Read('+:2.sps');
    Read('+:3.sps');
    Writep(a);
  end
end
```

57 Show Graph.Component (learn30)

—- Run LPL Code , HTML Document –

Problem: This model shows the use of the function Graph.Component. The model is an assignment problem, augmented with subtour elimination constraints of lengtn 2. Run the model and open the SVG graph learn30.svg.

The model also shows how to separate the main model from the data and from the output code. The model `data` and `output` are called automatically at an appropriate time of execution (`data` is called just before the minimization, and `output` is called at the very end). The user can change this by calling them explicitly at the intended place.

Note also that `output` is a `friend` of model `data` which means that the entities defined inside `data` are also available within the model `output` without explicitly noting the dot-notation (`data.X` for example).

Listing 57: The Main Model implemented in LPL [2]

```
model learn30 "Show Graph.Component";
  parameter n:=[30] "Graph Size";
  set i,j,k         "A set of vertices";
  parameter c{i,j}  "Distance between two locations i and j";
  parameter cNr1    "number of components";
            passed  "loop variable";
            C{i}    "the components";
  binary variable x{i,j|i<>j} "Is 1 if (i,j) is in the tour";
  constraint
    A{i}: sum{j} x = 1;
    B{j}: sum{i} x = 1;
    S2{i,j|j>i}: x[i,j] + x[j,i] <= 1; //no subtours of length 2
  minimize obj: sum{i,j} c*x;
  cNr1:=Graph.Components(x,C);
  Write('There␣are␣%d␣components\n',cNr1);
  while passed<cNr1 do
    passed:=passed+1;
    Write('\nComponent␣%d:␣',passed);
    Write{i|C=passed}('%2s␣',i);
  end
end
```

Listing 58: The Data Model

```
  model data;
    parameter X{i}; Y{i}; m:=Trunc(Sqrt(n));;
    i:=1..n;
    X{i}:=(i%m+1)*2+Trunc(Rnd(0,2));
    Y{i}:=(i/m+1)*2+Trunc(Rnd(0,2));
    c{i,j}:= Sqrt((X[j]-X[i])^2+(Y[j]-Y[i])^2);
  end
```

Listing 59: The Output Model

```
  model output friend data;
    Draw.Scale(30,30);
    Draw.DefFont('Verdana',8);
    for{i,j|x} do Draw.Line(X[i],Y[i],X[j],Y[j],3);  end;
    for{i} do Draw.Circle(i&'',X,Y,.3,1,0) ; end;
    Draw.Text('Length='&Round(obj,-4),0,0,16);
  end
```

58 Show Graph.Mincut function (learn31)

–– Run LPL Code , HTML Document –

Problem: The function `Graph.Mincut()` implements the Stoer-Wagner min cut algorithm. Unfortunately, this function has not yet been well tested, so no guarantee is given... (to be checked)

Listing 60: The Complete Model implemented in LPL [2]

```
model learn31 "Show Graph.Mincut function";
  set i,j;    --nodes
    e{i,j};   --edges
    y{i,j}; YY{i};
  parameter X{i}; Y{i};
  parameter u{i,j}; C;
  ;
  readgraph;
  C:=Graph.Mincut(u,y,YY,0);
  Write('Min␣Cut␣value␣is:␣%d\n', C);
  Write{i,j|y}('␣␣(%1s,%1s)\n', i,j);
  Write{i|YY}('␣%s␣', i);
  drawgraph;
  model readgraph;
    i:=[1..8];
    u{i,j}:=/1 2 2, 1 5 3, 2 3 3, 2 5 2, 2 6 2, 3 4 4, 3 7 2,
              4 7 2, 4 8 2, 5 6 3, 6 7 1, 7 8 3/;
    u{i,j}:=if(u,u,u[j,i]);
    e{i,j}:=if(i<j,u);
    X{i}:=[0 1 2 3 0 1 2 3];
    Y{i}:=[0 0 0 0 1 1 1 1];
  end
  model drawgraph;
    e[2,1]:=0;  //do nor draw
    Draw.Scale(100,100);
    Draw.DefFont('Verdana',8);
    for{e[i,j]} do Draw.Arrow('('&y&','&u&')',
      X[i],Y[i],X[j],Y[j],.2,if(y=u,3,y>0,4,0)); end
    for{i} do Draw.Circle(i&'',X,Y,.2,1,0); end
  end
end
```

59 Show Graph.MStree (Minimal Spanning Tree) (learn32)

—– Run LPL Code , HTML Document –

Problem: The function `Graph.MStree()` implements the Prim algorithm for the minimal spanning tree problem in a graph. (Note that the graph data are read for THIS file(!) directly.)

Listing 61: The Complete Model implemented in LPL [2]

```
model learn32 "Show Graph.MStree (Minimal Spanning Tree)";
  set i,j,k; e{i,j}; x{i,j};
  parameter X{i}; Y{i};
  parameter c{i,j|e}; len;
  ;
  readgraph;
  c{i,j}:=Sqrt((X[j]-X[i])^2+(Y[j]-Y[i])^2);
  len:=Graph.Mstree(c,x);
  drawgraph;
  model readgraph;
    string parameter FILE:=['learn32.lpl'];
    Read{i}(FILE&',%1:Coordinates', i,X,Y);
    Read{i,j}('%1:Edges', i,j,e,c);
    e{i,j}:=if(e,e,e[j,i]);
  end
  model drawgraph;
    e[2,1]:=0;
    Draw.Scale(60,60);
    Draw.DefFont('Verdana',8);
    for{e[i,j]|x} do Draw.Line(Round(c,-2)&'',X[i],Y[i],X[j],Y[j],3);
       end
    for{i} do Draw.Circle(i&'',X,Y,.2,1,0); end
  end
end
```

60 Read Multiple Snapshots (learn33)

–– Run LPL Code , HTML Document –

Problem: This model is a small example that shows the power of shapshot files. A snapshot file is a file that records ALL model data at the call of a Write snapshot instruction. Later on, this snapshot can be read in. The filename has extension `sps`. The following function call simply writes a snapshot file named `snap.sps`.

```
Write('snap.sps');
```

The instruction writes a snapshot of the complete internal data store of the LPL model at the moment of its call to a file – the snapshot file, that is, all data at a particular moment of the execution are collected and written to the file. This "data state" can be restored at any time by a snapshot `Read` instruction.

```
Read('snap.sps');
```

Note that data tables with the `frozen` attribute set are not modified.

Listing 62: The Complete Model implemented in LPL [2]

```
model learn33 "Read Multiple Snapshots";
  set i,j frozen :=[A B C D E];  --unchangable set
  set k :=[1..4];
  parameter a; b{i}; c{k,i};
  for{i} do
    a:=10*i;              --assign a value to a
    b{j}:=i*j;            --fill table b
    c{k,j}:=j*10*k+i;     --fill table c
    Write(i&'.sps');      --write snapfiles: A.sns, B.sns, ...
  end;
  ClearData(learn33);
  Read('D.sps');
  Write('\nRead␣the␣snapshot␣D.sps␣only:\n------------\n');
  Writep(a,b,c);
  ClearData(learn33);
  Write('\nRead␣all␣snapshots␣cummulatively:\n------------\n');
  for{i} do
    Read('+:'&i&'.sps');    --read snapfiles cummulative
  end
  Writep(a,b,c);
end
```

Snapshot files are particular interesting, when a lenghty optimization takes place: After the optimization, a snapshot can be written. At a later date, this snapshot can be read again and LPL's data store is in a state as if the optimization had taken place.

Several snapshots can be written to various files, as is done in the model above within the first loop. What is interesting: all the snapshot can be read individually or *cumulatively*. Cumulatively means, that

1. The LPL internal store is augmented with an additional index-set called `_SNAP_`.
2. All data tables are expanded by the new index-set, that is, a singleton becomes indexed, a vector becomes a two-dimensional table, etc.
3. The "slices" of the tables are cumulatively filled by each read snapshot.
4. At the end, we have *all* snapshots stored in parallel and the tables can be viewed as if they were ordinary tables – augmented by the snapshot index-set.

This allows the modeller to compare the different variants.

In the first loop of the model, the parameter `a` is modified to become: 10, 20, ... , 50, the tables `b` and `c` are filled with different values and the snapshot files `A.sps` ... `E.sps` are written.

Then all data are cleared, with the exception of the set `i` which has the attribute `frozen`. Then just one particular snapshot is read – the fourth (`D.sps`). And the data are written to control LPL's data.

The data are again cleared and in the second loop, all snapshot files are read with the prefix reading instruction `+:` , which means to read the snapshots cumulatively. All data are again written to view LPL's internal store.

61 Freeze function (learn34)

–– Run LPL Code , HTML Document –

Problem: This model is a small example that shows the use of the indexed `Freeze` function. "Freezing" a variable means to fix its preset value and they are sent as fix bounds to the solver. Of course, LPL can change there value at any time using an assignment.

"Freezing a constraint" means to drop a constraint such that the solver does not see it. Variables and constraints can be "frozen" and "unfrozen" at any time using `Freeze` and `Unfreeze`.

Listing 63: The Complete Model implemented in LPL [2]

```
model learn34 "Freeze function";
  set
    i := [1..4];
    j := [1..5];
    c1{i} := i <= 3;;
  binary variable
    x{i,j};
    p_C1{c1[i]};;
  p_C1{c1[i]} := 0;          // Assign three variables
  Freeze({i|i=1} p_C1[i]);   // Freeze variable p_C1[1] to 0
  constraint
    C1{c1[i]} : sum{j} x[i,j] <= 1 + p_C1[i];
  Freeze({i|i=2} C1[i]);     // Freeze a constraint
  maximize Obj: sum{i,j} x[i,j];
  Write('Obj:␣%f\n', Obj);
end
```

The model above defines 3 variables `p_C1` – for the first three indexes in `i`. The following instructuion fixes the first variable to zero:

```
     Freeze({i|i=1} p_C1[i]);
```

This variable is sent to the solver as a fix-bounded variable.
The model also declares three constraints `C1` of which the second is "frozen" with the instruction

```
     Freeze({i|i=2} C1[i]);
```

To see the effect of these two instructions, the user must generate the EQU-file. LPL only generates the two following constraints

```
C1[1]:  - p_C1[1] + x[1,1] + x[1,2] + x[1,3] + x[1,4] + x[1,5] <= 1;
C1[3]:  - p_C1[3] + x[3,1] + x[3,2] + x[3,3] + x[3,4] + x[3,5] <= 1;
```

One can also see that the value of `p_C1[1` is 0 as it was frosen to that value.

62 function En(), El() (learn35)

–– Run LPL Code , HTML Document –

Problem: All elements of a set in LPL have a ”dual nature”. The elements are first of all strings, even if some have numerical values (with the exception 1..n – which are only numbers). But since all sets in LPL are ordered, each element also has a *position*. The first element of a set has position 1 by default, the second has position 2, and so on.

The function $El(i,e)$ retruns the position in set i of element e, e must be an element name – hence a string. If e is an index name it is automatically interpreted as a string.

The function $En(i,e)$ returns the element name in set i, e must be the position of e within i – hence a number. If e is an index name then it is automatically interpreted as a number.

Listing 64: The Complete Model implemented in LPL [2]

```
model learn35 "function En(), El()";
  set i:=[Aa Bb Cc Dd Ee Ff];
  SetFocus();
  NextFocus(0);
  while NextPosition do  Write('␣␣%s', GetValueS(0)); end
  Write('\nThe␣set␣i␣is:␣%3s\n', {i} i);
  Write('The␣third␣element␣in␣i␣is:␣%s\n', En(i,3));
  Write('Dd␣is␣at␣position␣%d␣in␣set␣i\n', El(i,'Dd'));
  Write{i}('%s␣is␣at␣position␣%d\n',i, El(i,i));
  Write{i}('The␣%d-th␣position␣is␣element␣%s\n',i, En(i,i));
end
```

63 Show Graph.Bfs (learn36)

—- Run LPL Code , HTML Document –

Problem: This model shows the use of the function Graph.Bfs() which implements the depth first search of a graph. The instruction in the model `Graph.Bfs(e,p,0);` explores the component starting at node '0'. The search tree is the subgraph with the red edges. A more interesting example is given in model maze[19].

Listing 65: The Complete Model implemented in LPL [2]

```
model learn36 "Show Graph.Bfs";
  parameter n:=[20] "Graph Size";
  set i,j           "A set of vertices";
      e{i,j}        "A edge list";
      t{i,j}        "tree edges";
  parameter p{i}    "the search tree";
  Graph.Bfs(e,p,0);
  {i|p[i]<>-1} (t[i,p[i]]:=1, t[p[i],i]:=1);
  model data;
    parameter X{i}; Y{i}; m:=Trunc(Sqrt(n));;
    i:=0..n-1;
    X{i}:=(i%m+1)*2+Trunc(Rnd(0,2));
    Y{i}:=(i/m+1)*2+Trunc(Rnd(0,2));
    {i,j} if(Rnd(0,1)<=.03, (e[i,j]:=1, e[j,i]:=1));
  end
  model output friend data;
    Draw.Scale(30,30);
    Draw.DefFont('Verdana',8);
    {e[i,j]} Draw.Line(X[i],Y[i],X[j],Y[j]);
    {t[i,j]} Draw.Line(X[i],Y[i],X[j],Y[j],3,2);
    {i} Draw.Circle(i&'',X,Y,.2,1,0);
  end
end
```

[19]https://lpl.matmod.ch/lpl/Solver.jsp?name=/maze

64 Show Graph.SPath (shortest path) (learn37)

—- Run LPL Code , HTML Document –

Problem: The function `Graph.SPath()` implement the Belman-Ford algorithm for the shortest path problem.

Listing 66: The Complete Model implemented in LPL [2]

```
model learn37 "Show Graph.SPath (shortest path)";
  set i,j,k; e{i,j}; x{i,j};
  parameter X{i}; Y{i};
  parameter c{i,j|e}; len;
  ;
  readgraph;
  c{i,j}:=Sqrt((X[j]-X[i])^2+(Y[j]-Y[i])^2);
  len:=Graph.SPath(c,x,1);
  drawgraph;
  model readgraph;
    string parameter FILE:=['learn37.lpl'];
    Read{i}(FILE&',%1:Coordinates', i,X,Y);
    Read{i,j}('%1:Edges', i,j,e,c);
    e{i,j}:=if(e,e,e[j,i]);
  end
  model drawgraph;
    e[2,1]:=0;
    Draw.Scale(60,60);
    Draw.DefFont('Verdana',8);
    {e[i,j]|x} Draw.Line(Round(c,-2)&'',X[i],Y[i],X[j],Y[j],3);
    {i} Draw.Circle(i&'',X,Y,.2,1,0);
  end
end
```

65 Sl function (goal programming) (learn39)

—– Run LPL Code , HTML Document –

Problem: The function `Sl(a,b)` can be used to model "soft" constraints (goal programming), that is, it introduces slack variables automatically and penalizes them in the objective function. "`a`" is an expression that models the upper bound of the slack variable (the lower bound is 0). "`b`" is an expression for the weight in the objective function. To explain the different usages, this model gives same hints.

Listing 67: The Complete Model implemented in LPL [2]

```
model learn39 "Sl function (goal programming)";
  set i:=1..3; j:=1..5; k:=1..3;
  variable x{i}; y{i}; z{j}; w [0..0]; v;
  parameter a{i}:=10*i;  b{i}:=100*i;
  parameter cc{i,k}:=k; ww{i,k}:=10*k;
  constraint A{i}: w+x+y+sum{j} z >= 1 - Sl(a>11,a) - Sl(3,b);
  constraint B{i|i<3}: x >= 2*i - Sl(2,0.9);
  constraint C{i|i>1}: sum{j} z >= 1 - Sl{k|k<=2}(cc,ww);
  constraint D{i|i>1}: sum{j} z >= 1 - sum{k|k<=2} Sl(cc,ww);
  minimize obj: sum{i} x;
  Write('The␣slacks␣of␣B␣are:␣%5.1f\n', {i|i<3} GetValue(B,9));
  Write('Because:␣The␣values␣of␣the␣left␣hand␣side␣(x)␣are␣0␣2,␣the␣
     right␣hand␣side␣are:␣2␣4\n');
end
```

In the constraint `A` the function `Sl(a>11,10*i)` is used. It adds a new indexed variable to the model as follows:

```
variable A_x{i} [0..a>11];
```

The variable name is the corresponding constraint name plus `'_x'`. The upper bounds of these variables are `a[i]>11` (which is 0 for a_1 and 1 for a_2 and for a_3 in this case). Slack variables with an upper bound of 0 (since the lower bound is also 0) are eliminated automatically from the model. (Note: other variables with a lower and upper bound of zero, like `w` are *not* eliminated from the model.) In addition, the following terms are added to the objective function `obj`:

```
... + 20*A_x[2] + 30*A_x[3] + ...
```

Also in the constraint `A` a second function `Sl(3,b)` is used[20]. It adds a second slack variable to the model with the fixed upper bound of 3 and a weight of b_i to the objective function:

```
variable A_x1{i} [0..3]
```

The variable name is the corresponding constraint name plus `'_x1'`. (If several `Sl` functions occur in a constraint, then the corresponding additional slack variables name contains an additional number $1, 2, \ldots$.) It also adds the following terms to the objective function:

```
... +100*A_x1[1] +200*A_x1[2] +300*A_x1[3]
```

The constraint `B` displays the sparsity aspect. Since `B` is only defined for $i \leq 2$ the corresponding slack variables `B_x{i}` are also generated with the same sparsity.

The constraint `C` and `D` are exactly the same. The `Sl`-function can be indexed and this is nothing else than "syntax-sugar" and is translated into the constraint `D` where the index-list is removed from the `Sl` and placed in a sum operator. The slack variables are indexed correspondingly. In the model case, the variable definitions are:

[20]Note that it makes perfectly sense in real applications to used several slacks with different upper bounds and penalties (weights) in the objectives. This allows the modeller to model piece-wise linear penalties for various goals.

```
C_x{i,k|i>1 and k<=2} [0..cc];
D_x{i,k|i>1 and k<=2} [0..cc];
```

The function `GetValue(B,9)` returns the value of the corresponding slack variable. Note also that the compiler switch ’p’ adds a `Sl` function term to each constraint. LPL generates the following EQU-file (equation listing) :

```
min: +10 D_x[1,2] +10 D_x[1,3] +20 D_x[2,2] +20 D_x[2,3] +10 C_x[1,2] +10 C_x[1,3]
  +20 C_x[2,2] +20 C_x[2,3] +0.9 B_x[1] +0.9 B_x[2] +100 A_x1[1] +200 A_x1[2]
  +300 A_x1[3] +20 A_x[2] +30 A_x[3] +x[1] +x[2] +x[3];
A[1]:  +A_x1[1] +z[1] +z[2] +z[3] +z[4] +z[5] +y[1] +x[1] +w >= 1;
A[2]:  +A_x1[2] +A_x[2] +z[1] +z[2] +z[3] +z[4] +z[5] +y[2] +x[2] +w >= 1;
A[3]:  +A_x1[3] +A_x[3] +z[1] +z[2] +z[3] +z[4] +z[5] +y[3] +x[3] +w >= 1;
B[1]:  +B_x[1] +x[1] >= 2;
B[2]:  +B_x[2] +x[2] >= 4;
C[2]:  +C_x[1,2] +C_x[2,2] +z[1] +z[2] +z[3] +z[4] +z[5] >= 1;
C[3]:  +C_x[1,3] +C_x[2,3] +z[1] +z[2] +z[3] +z[4] +z[5] >= 1;
D[2]:  +D_x[1,2] +D_x[2,2] +z[1] +z[2] +z[3] +z[4] +z[5] >= 1;
D[3]:  +D_x[1,3] +D_x[2,3] +z[1] +z[2] +z[3] +z[4] +z[5] >= 1;
```

Note: the `Sl` function can also be added in an objective function, in this case, it implements a pre-emptive goal programming, see model library[21] and compare it with model library-1[22].

[21] https://lpl.matmod.ch/lpl/Solver.jsp?name=/library

[22] https://lpl.matmod.ch/lpl/Solver.jsp?name=/library-1

66 Parameterized Calling of Submodels (learn40)

–— Run LPL Code , HTML Document –

Problem: This model shows how one can generated several model 'variants' using the **subject_to** attribute. The variants depend on the parameter **var**. For example, if `var='case2'` then the model first runs the submodel `case2` then it continues running the main model which minimizes a function subjected to the constraints defined in main model `learn40` and in addition in the submodel `addConst0`.

Listing 68: The Complete Model implemented in LPL [2]

```
model learn40 "Parameterized Calling of Submodels";
    var:='case2',
    if(var='case1', case1,
       var='case2', case2,
       var='case3', case3,
       var='case4', case4);
  //or alternatively: RunMain;
  --------- end main run: all declarations begin here
      ------------------------------
  --------- note: declarations can be in ANY order
      ---------------------------------
  string parameter var; //parameter to choose "variant"
  set i;
  parameter a{i}; c{i}; B;
  integer variable x{i};
  constraint C: sum{i} a*x = B;
  minimize obj: sum{i} c*x
  subject_to
    if(var='case1',learn40,
       var='case2',(learn40,addConst0),
       var='case3',(learn40,addConst0),
       var='case4',learn40);
  Writep(x);
/*  alternatively: this main model to run
  model RunMain;
    var:='case2';
    if var='case1' then case1; output; end;
    if var='case2' then case2; end;
    if var='case3' then case3; end;
    if var='case4' then case4; end;
  end
*/
  ----- the model "variants"
  model case1;
    myData0; aPlus10;
  end
  model case2;
    Writep(var);
    myData; addConst0;
  end
  model case3;
    myData; aPlus10; addConst0;
  end
  model case4;
    myData; minItems;
  end
  ----- several variations
  model myData0;
```

```
    i := [1..10];
    a{i} := [3 4 5 2 6 7 4 5 3 9];
    c{i} := [2 3 5 7 9 8 6 4 2 1];
    B := 233;
  end
  model myData "A model data (defining the base case)";
    i := [ chocolate milk apple water mais chips tomatoes beer wine
       tabacco ];
    a{i} := [3 4 5 2 6 7 4 5 3 9];
    c{i} := [2 3 5 7 9 8 6 4 2 1];
    B := 233;
  end
  model aPlus10 "Augment a by 1";
    a{i} := a+1;
  end
  model addConst0 "Exactly 30 items";
    constraint A: sum{i} x = 30;
  end
  model minItems "Minimize the number of items";
    Freeze(obj); //replace objective function !!!
    minimize minI: sum{i} x subject_to learn40;
  end
  model output;
    Write('Capacity␣is:␣%d\n', B);
  end
end
```

67 A Model with Parameters I (learn41)

—– Run LPL Code , HTML Document –

Problem: A model in LPL is like a function – by the way `model` and `function` are exchangable keywords – that can be called and can return a value. The first example is `mymodel`, a parameterless model that returns 4.

```
model mymodel;
  return 2*2;
end
```

Models definition can also contain formal parameters like `isSlotEarly`. This model calls itself other functions and returns a Boolean value. Note that recursive calls are not possible in LPL.

```
function isSlotEarly(integer _d1; _d2);
  return isSlotE(_d1,_d2) or isSlotSE(_d1,_d2)
           or isSlotE1(_d1,_d2);
end
```

A Third example is the function `Euclid` which returns the greatest common dividor of two positive integers.

```
model Euclid(integer a;b);
  integer t;
  return while(b>0,(t:=a, a:=b, b:=t%b, a));
end
```

Common to all these functions is that the formal parameters must be declared as parameters and are singleton (not indexed). In this case, the parameters are passed as value (not as reference). Model learn42[23] shows an example where parameters are passed by reference.

Listing 69: The Complete Model implemented in LPL [2]

```
model learn41 "A Model with Parameters I";
  parameter e_StartE :=10; e_EndE:=100;
  parameter e_StartSE:=11; e_EndSE:=90;
  parameter e_StartE1:=12; e_EndE1:=80;
  function isSlotE(integer _d1; _d2);
    return _d1 >= e_StartE  and _d2 <= e_EndE  and _d1 <= _d2;
  end
  function isSlotSE(integer _d1; _d2);
    return _d1 >= e_StartE  and _d2 <= e_EndE  and _d1 <= _d2;
  end
  function isSlotE1(integer _d1;  _d2);
    return _d1 >= e_StartE  and _d2 <= e_EndE  and _d1 <= _d2;
  end
  function isSlotEarly(integer _d1; _d2);
    return isSlotE(_d1,_d2) or isSlotSE(_d1,_d2)
             or isSlotE1(_d1,_d2);
  end
  model Euclid(integer a;b);
    integer t;
    return while(b>0,(t:=a, a:=b, b:=t%b, a));
  end
  model mymodel;
    return 2*2;
  end
  Write('\'mymodel\'␣called,␣returns␣:␣%d\n', mymodel);
```

[23]https://lpl.matmod.ch/lpl/Solver.jsp?name=/learn42

```
  Write('%b\n', isSlotEarly(20,110));
  Write('gcd␣of␣994009␣and␣96709␣is␣:␣%d\n', Euclid(994009,96709));
end
```

68 A Model with Parameters II (learn42)

–– Run LPL Code , HTML Document –

Problem: This model shows submodels (functions) with argumants. It contains two sobmodels mult2 that returns a value multiplied by 2. The formal parameter is an expression or a singleton parameter. In this case, they are *passed by value*.
The second submodel knapsack is a complete knapsack model with 5 formal parameters being passed. It encapsulates a complete optimisation model and return the objective value. 4 of the five parameters are *passed by reference* and one is passed by value (K) because it is a singleton parameter. When calling the model the parameters must exactly match: a set must be called as a set, a varibale must be called as a variable, even the type must match: If a formal parameter is defined as an integer variable then it must also be called as an integer variable.
The third example is the model returnAsIs. The argument x is a singleton value, so it must be passed by value. However the call of the function contains a indexed parameter, but since it is passed as A[j] it is considered as an expression and hence pass by value.

Listing 70: The Complete Model implemented in LPL [2]

```
model learn42 "A Model with Parameters II";
  integer x:=2;
  integer y:=mult2(x+1); //pass by value
  integer z:=mult2(4);   //pass by value
  Writep(y,z);
  --------------------------
  set j:=1..10;
  parameter A{j} := Trunc(Rnd(3,7));
  parameter B{j} := Trunc(Rnd(30,50));
  integer variable X{j};
  parameter KK:=40;
  parameter obj:=knapsack(j,KK,A,B,X); //pass by ref and value
  Writep(obj,X);
  ----------------------------
  Write{j}('%d\n', returnAsIs(A[j])); //pass by value
  ----------------------------
  integer model mult2(integer n);
    return 2*n;
  end;
  real model knapsack(set i; parameter K; a{i}; b{i}; integer variable
     x{i});
    constraint A: sum{i} a*x <= K;
    maximize obj: sum{i} b*x;
    return obj;
  end;
  model returnAsIs(integer x);
    return x;
  end
end
```

69 Pivot Table (learn43)

—- Run LPL Code , HTML Document –

Problem: Download the model and open the 4-dimensional table *a* in lplw.exe and play around.

Listing 71: The Complete Model implemented in LPL [2]

```
model learn43 "Pivot Table";
  set i; j; k; h;
  parameter a{i,j,k,h};
  model data;
     i:=[1,2];
     j:=[1 2 3];
     k:=[1 2 3 4];
     h:=[1 2 3 4 5];
     a{i,j,k,h}:=if(i=1 and j=2 and k=3 and h=4,0,i*j*k*h));
  end
end
```

70 Colored Report (learn44)

—– Run LPL Code , HTML Document –

Problem: (This model supposes that you understand how FastReport works.) This model shows a report that colors the cells depending on their values: positive value are blue, negative values are red.

The first Write creates a new database and adds a table titleP with the unique field 'titel' and a unique row with the entry 'Titel'. Furthermore it will create two ReportBands: a `TfrxReportTitle` with a unique `TfrxMemoView` named `titleP_titel`, and a `TfrxPageFooter` (Option '0').

The second Write adds adatabas etable named tableH with 5 fields named b, a1,a2,a3,a4. Their content is ' ' (a blank), and the elements of p (2007 2008 2009 2010). Furthermore, it adds a ReportBand `TfrxPageHeader` with the corresponding fields.

The third Write adds a table ReportColor and adds 9 fields. It will also add a ReportBand `TfrxMasterData`. Finally it generates the report template `ReportColor.fr3` (if it does not exist). Then it creates a pdf file of the entire report.

If the report template `ReportColor.fr3` has been created then add the following code to the code page of ReportColor.fr3 :

```
function  rgb(r,g,b:int):int;
var c:int;
begin c:=trunc(b); c:=c shl 8 + trunc(g); c:=c shl 8 + trunc(r);
  result:=c;
end;

function col(Sender: TfrxMemoView) : int;
var d,a: double; x:int;
begin
  d := 150;
  a := Sender.Value;
  if (a<=511) and (a>=-511) then x:=trunc(d*abs(a)) else x:=511;
  if a<0 then begin if x>255 then x:=rgb(511-x,0,0) else x:=rgb(255,255-x,255-x); end
  else begin if x>255 then x:=rgb(0,0,511-x) else x:=rgb(255-x,255-x,255); end;
  //if (a<0) then Sender.Font.Color:=clWhite;
  Result:=x;
end;

procedure EvalOnAfterData(Sender: TfrxMemoView);
begin
  Sender.Color:=col(Sender);
end;

begin

end.
```

Then dd the function `EvalOnafterData` as an event `OnafterData` to the four MemoViews `ReportColor_a1` .. `ReportColor_a4`.

71 Circular Time Lag Operator (learn45)

—- Run LPL Code , HTML Document –

Problem: Since a set in LPL is always ordered, we can define circular relations (first element follows the last one).

Listing 72: The Complete Model implemented in LPL [2]

```
model learn45 "Circular Time Lag Operator";
  set  i  "periods" := [1 2 3 4 5 6 7 8 9 10 11 12];
  parameter  s{i} "number of desks open in a period"
        := [ 10 10 8 8 14 14 5 5 3 3 8 8 ];
    t{i} :=
        [ 9.2  8.8  8.8  8.4  8.0  8.0  8.4  8.8  8.8  9.2  9.6  9.6 ];
  variable  x{i} "number of clercs beginning in periode i";
  constraint
     r{i}: x + x[i%#i+1] + x[(i+2)%#i+1] + x[(i+3)%#i+1] >= s;
  minimize z: sum{i} t*x;
  Writep(z,x);
end
```

72 Function xP (learn46)

—- Run LPL Code , HTML Document –

Problem: The function xP(C,n) return the position of the n-th index. In the example a matrix table $c_{,j}$ von 10×10 with random values between 1 and 100 is created. The values are sorted and the result is stored in the permutation C. C holds the position of the values, but the position is a single integer, counting the entries in a lexicographical way. so if C[i,j] is 46, for example then xP(C,1) is 5 and xP(C,2) is 6, defining the entry $(5,6)$ in the $c_{i,j}$ table.

Listing 73: The Complete Model implemented in LPL [2]

```
model learn46 "Function xP";
  set i,j := 1..10;
  parameter C{i,j}; c{i,j}:=Trunc(Rnd(1,100));
  Sort(c,C);
  Write{i,j}('%2d␣%2d␣%3d␣%5.2f\n', xP(C,1),xP(C,2),C,c[C]);
end
```

73 A logical constraint (learn47)

—- Run LPL Code , HTML Document –

Problem:

Listing 74: The Complete Model implemented in LPL [2]

```
model learn47 "A logical constraint";
  set i:=1..5;
  binary variable y; x{i};
  constraint
    AND: y = and{i} x;
  maximize obj: y;
end
```

74 Run some code inside a constraint (learn48)

—– Run LPL Code , HTML Document –

Problem: Just before a constraint is generated, one can run some code to create data for that constraint. The code `(n:=i , m:=i^2)` is run just before the constraint is generated in this model:

Listing 75: The Complete Model implemented in LPL [2]

```
model learn48 "Run some code inside a constraint";
  set i:= 1..10;
  parameter n; m;
  variable x{i}; y;
  constraint A{i}: (n:=i , m:=i^2) , x[i]+y = n+m; //cool feature
  minimize obj: y;
end
```

It is important to put the code inside parentheses if more then one expression (assignment) is to be executed before the constraint. Furthermore, the code cannot contain variables, and the actual constraint expression (here `x[i]+y = n+m`) must follow after the comma.

This feature could also be used to run some output to debug the constraint generation as in:

```
  parameter R{i};
  ....
  constraint A{i}: Write('+:file.txt','%3d\n',n) , x[i]+y = R[i];
```

75 Queue and Map data structure (learn49)

—- Run LPL Code , HTML Document –

Problem: Create Queue mit 10 entries, EnQueue the number 1..5 then DeQueue them. Next: Add 5 elements (1..5) to a Map then check whether the number 1..10 are in the Map. For an interesting model see rushhour[24].

Listing 76: The Complete Model implemented in LPL [2]

```
model StructTest "Queue and Map data structure";
  set i:=1..5;
  Struct.Queue(10);
  {i} Struct.EnQueue(i);
  Write{i}('%d\n',Struct.DeQueue());
  set j:=1..10;
  {i} Struct.AddMap(i);
  Write{j}('%b\n',Struct.InMap(j));
  //free memory of Queue and Hash (Map) is automatic
end
```

[24]https://lpl.matmod.ch/lpl/Solver.jsp?name=/rushhour

76 Greatest Common Divider (learn51)

—- Run LPL Code , HTML Document –

Problem: This model implements a simple (inefficient) algorithm for finding the greatest common dividor (gcd) of two numbers. It loops through all numbers `i` from 1 to the smaller of both integers and check if both numbers are divisible by `i`. If this is the case it stores it (in `d`). The last encountered number is then the gcd.

Listing 77: The Complete Model implemented in LPL [2]

```
model gcd "Greatest Common Divider";
  integer a := 1943;  b := 2813;
    c := if(a<b, a, b);
    d := 1;
  for{i in 1..c} do
    if a%i = 0 and  b%i = 0 then
      d:=i;
    end
  end
  Write('The␣gcd␣of␣%d␣and␣%d␣is␣%d\n', a, b, d);
end
```

77 Topological Sorting (learn52)

–– Run LPL Code , HTML Document –

Problem: The function `Topo()` returns a topological sorting of the nodes of a DAG (a Directed Acyclic Graph). As one can see from the Figure 10, the graph is a DAG (it does not contain a cycle) and there is a unique topological sorting of the nodes (red).

Listing 78: The Complete Model implemented in LPL [2]

```
model Topo "Topological Sorting";
  set i,j:=[1..5] "nodes";
  set dag{i,j}:=
    [(2,1) (3,1) (3,2) (3,4) (4,1) (4,2) (5,1)
     (5,2) (5,3) (5,4)]  "arc list of a DAG";
  model output;
    parameter Z{i};
    Graph.Topo(dag,Z);
    Write('Sequence=%3d\n', {i} Z);
    parameter r:=#i*3; PI:=3.14159;
      X{i}:=r+r*Sin(2*PI*(i-1)/#i);
      Y{i}:=r+r*Cos(2*PI*(i-1)/#i);
    Draw.Scale(10,10);
    Draw.DefFont('Verdana',12);
    {i,j|dag} Draw.Arrow(X[i],Y[i],X[j],Y[j],2,0,1);
    {i|i<#i} Draw.Arrow(X[Z[i]],Y[Z[i]],X[Z[i+1]],Y[Z[i+1]],2,3,4);
    {i} Draw.Circle(i&'',X,Y,2,1,0);
  end
end
```

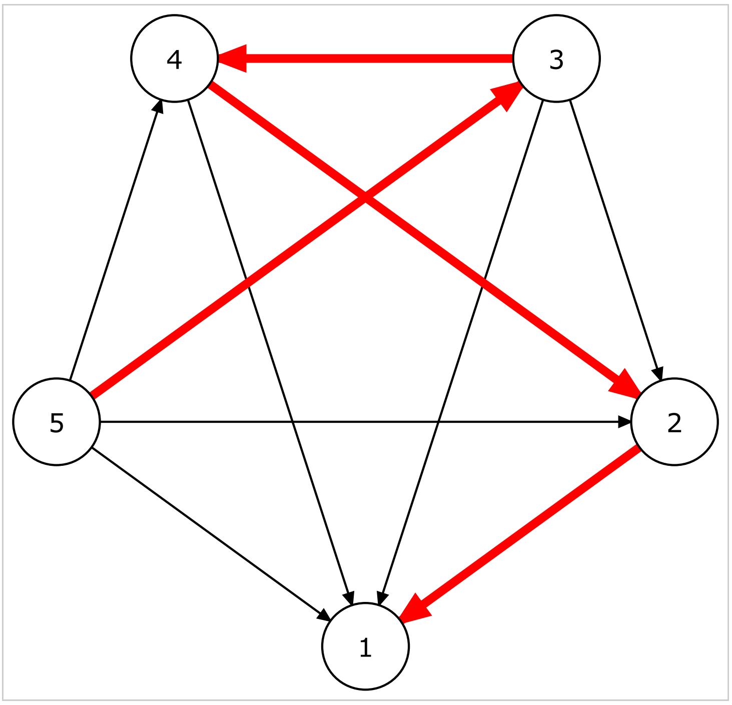


Figure 10: The Solution DAG annd the its Longest Path (red)

78 Reading various data sets (learn53)

—- Run LPL Code , HTML Document –

Problem: The model contain two data sets: one for a very small model instance, the other for a larger model. One can call either. There are two methods: One method to switch various data sets for the same model is to specify a parameter (here `selectData`) and depending on its value to choose the data set (either `data1` or `data2`). By default here the data set `data1` ia chosen. If one wants to run the data set `data2` instead – without changing the code – then LPL needs to be called as follows:

```
lplc learn53 - selectData=2
```

Another method is to store the data sets in different files – then one would choose the file in the APL (applied parameter list). For that the two following lines in the model must be removed :

```
parameter selectData := [1];
...
if selectData=1 then data1; else data2; end;
```

And LPL must be called by

```
lplc learn53 - @IN=data2
```

Note: `IN` is a fixed APL parameter to a link for the `data` file.

Listing 79: The Complete Model implemented in LPL [2]

```
model learn53 "Reading various data sets";
  parameter selectData := [1];
  set i; j;
  parameter a{i,j}; c{j}; b{i};
  variable x{j};
  constraint C{i}: sum{j} a[i,j]*x[j] <= b[i];
  if selectData=1 then data1; else data2; end;
  maximize Obj: sum{j} c[j]*x[j];
  Write('Objective Value = %7.2f \n', Obj);
  Write{j|x}('   x%-4s = %6.2f\n' , j,x);
  model data1;
    i := [1 2];
    j := [a b];
    b{i} := [350 300];
    c{j} := [300 200];
    a{i,j} := [5 5 , 6 2];
  end;
  model data2;
    parameter m := 1000;  n := 2000;;
    i := 1..m;  j := 1..n;
    a{i,j} := if(Rnd(0,1)<0.02 , Rnd(0,60));
    c{j}   := if(Rnd(0,1)<0.87 , Rnd(0,9));
    b{i}   := if(Rnd(0,1)<0.87 , Rnd(10,70000));
  end;
end
```

79 How to call a OS function (learn54)

—- Run LPL Code , HTML Document –

Problem: A operating system function or a program can be called from LPL.A example is:

Listing 80: The Complete Model implemented in LPL [2]

```
model learn54 "How to call a OS function";
  //OSCall('notepad.exe');  // calling notepad
  OSCall('cmd␣/c␣mkdir␣xyz'); //creates a directory called xyz
 end
```

80 Draw Lines/Opacity (xDrawAll)

—– Run LPL Code , HTML Document –

Problem:

Listing 81: The Complete Model implemented in LPL [2]

```
model xDrawAll "Draw Lines/Opacity";
  Draw.DefFont('Verdana,sans-serif',13,-2,2);
  Draw.DefFill('rect',Rgb(0,255,0));
  Draw.DefFill('circle',Rgb(255,255,51),Rgb(180,180,180),10);
  Draw.DefFill('circle.d',-1,-2,1);
  Draw.DefFill('path',Rgb(0,255,0),0);
  Draw.DefFill('path.d',-1,Rgb(0,255,0),10);
  Draw.DefLine('path.e',-2,-2,-2,-2,3);
  Draw.DefLine('path.f',-2,-2,-2,-2,2);
  Draw.DefLine('path.g',-2,-2,-2,-2,1);
  Draw.DefFill('line.x',-1,Rgb(0,255,0),20);
  Draw.DefFill('line.d',-1,Rgb(0,255,0),2);
  Draw.DefLine('line.r',-2,-2,-2,2);
  Draw.DefLine('line.s',-2,-2,-2,3);
  Draw.DefLine('line.dash',8,3);
  set i:=[1..5];
  parameter o1{i}:=[1 .2 .8 .5 1];
            o2{i}:=[1 .2 .8 1 .5];
  string t{i}:=['normal','opacity␣.2','opacity␣.8','fill-opacity␣.5','
     stroke-opacity.5'];
  for{i in 1..5} do
    Draw.Rect(20+140*(i-1),30,120,60);
    Draw.Circle(80+140*(i-1),60,50,-2,-2,-2,o1,o2);
    Draw.Text(t,22+140*(i-1),130);
  end;
  set j:=[1..3]; string parameter t1{j}:=['the␣form','fill-rule:nonzero
     ','fill-rule:evenodd'];
  parameter px{i}:=[120,-90,30,30,-90];
            py{i}:=[  0,100,-70,70,-100];
  for{j in 1..3} do
    Draw.Path('M',20+200*(j-1),150);
    for{i} do Draw.Path('l',px,py); end;
    Draw.Path('Y',if(j=1,-1,-2),-2,-2,-2,-2,if(j=3,2,-2));
    Draw.Text(t1,100+200*(j-1),210);
  end;
  Draw.Line('#x',30,400,30,300);
  Draw.Line('#r␣x',80,400,80,300);    //stroke-linecap:round
  Draw.Line('#s␣x',130,400,130,300);  //stroke-linecap:square
  Draw.Line(10,300,163,300,0,1);
  Draw.Line(10,400,163,400,0,1);
  Draw.Text('stroke-linecap',20,430);
  Draw.Text('butt',15,340);
  Draw.Text('round',65,360);
  Draw.Text('square',115,380);
  Draw.Path('M',180,400); Draw.Path('l',120,-50); Draw.Path('l'
     ,-70,100);
  Draw.Path('#d␣e','Y');
  Draw.Text('stroke-linejoin:round',240,440);
  Draw.Path('M',220,370); Draw.Path('l',120,-50); Draw.Path('l'
     ,-70,100);
  Draw.Path('#d␣f','Y');
  Draw.Text('stroke-linejoin:bevel',280,410);
  Draw.Path('M',260,340); Draw.Path('l',120,-50); Draw.Path('l'
      ,-70,100);
  Draw.Path('#d␣g','Y');
  Draw.Text('stroke-linejoin:miter',320,380);
  Draw.Line('#dash',690,460,590,265,2,3);
end
```

Output as follows:

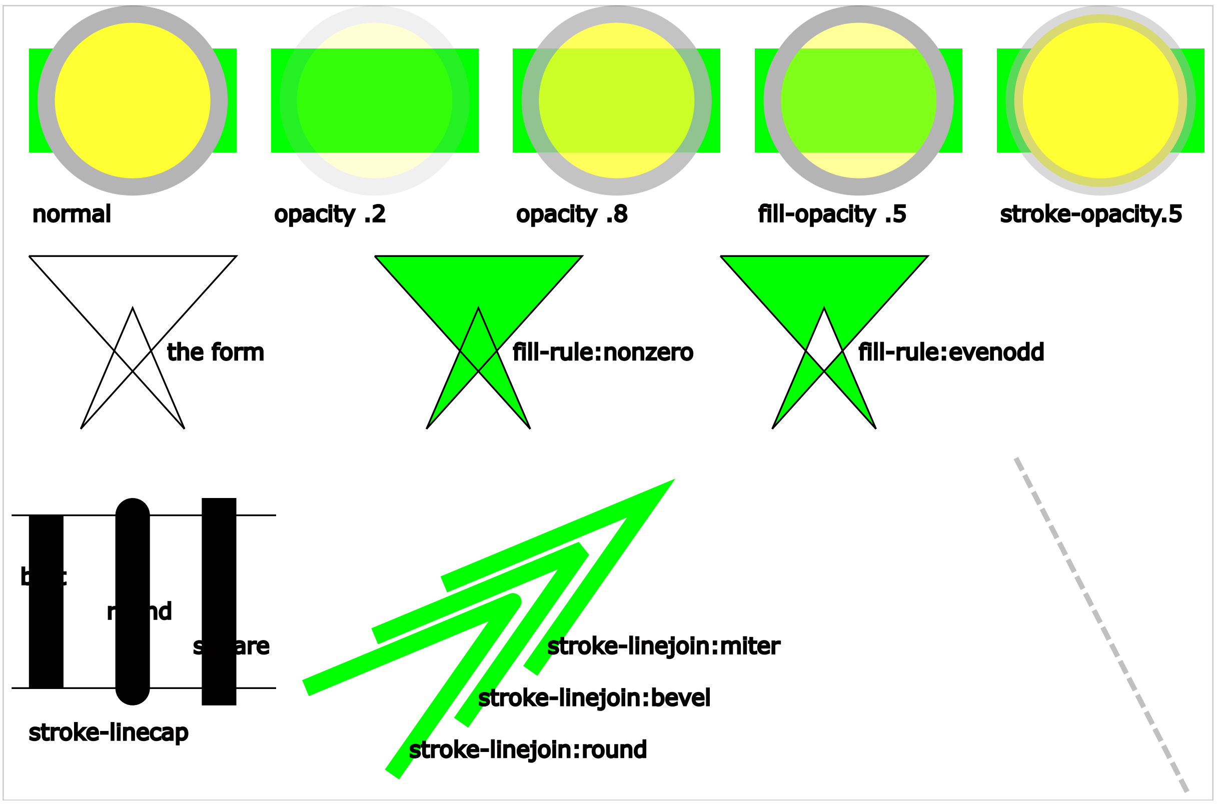


Figure 11: Output

81 Use arc path (xDrawArcPath)

–– Run LPL Code , HTML Document –

Problem:

Listing 82: The Complete Model implemented in LPL [2]

```
model xDrawArcPath "Use arc path";
  Draw.Scale(2,2);
  --Draw.Scale(2,-2);
  Draw.Path('M',20,255);
  Draw.Path('L',100,255);
  Draw.Path('A',10,70,90,1,1,180,255);
  Draw.Path('A',70,100,70,1,0,260,255);
  Draw.Path('L',370,255);
  Draw.Path('Y',-1,0);
  Draw.Circle(20,255,3);
  Draw.Circle(100,255,3);
  Draw.Circle(180,255,3);
  Draw.Circle(260,255,3);
  Draw.Circle(370,255,3);
end
```

Output as follows:

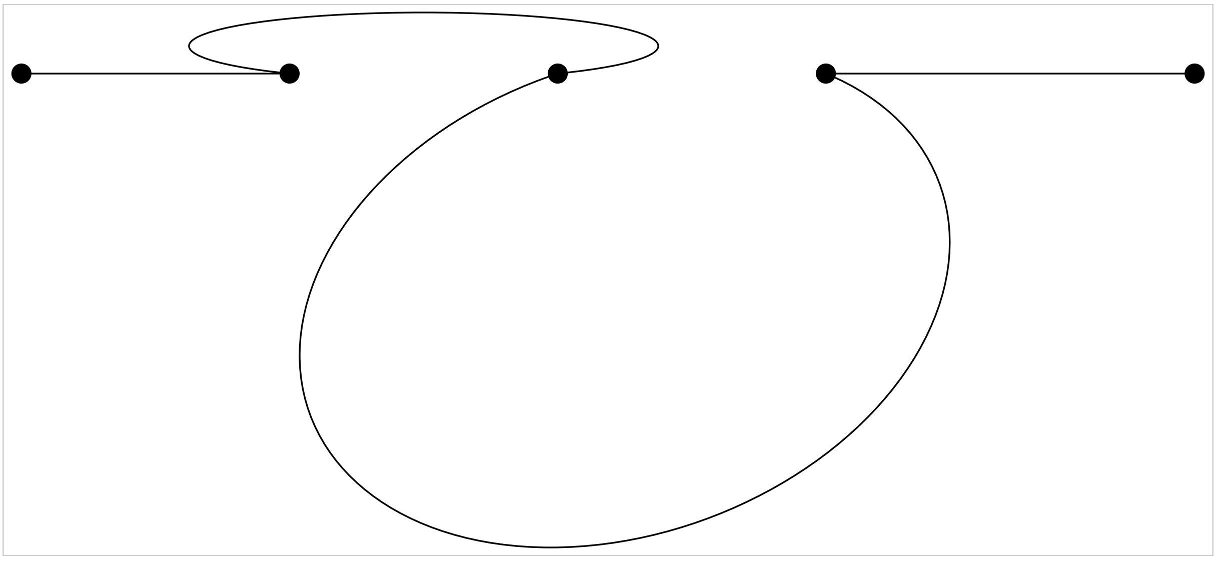


Figure 12: Output

82 Drawing Circles (xDrawCircle)

—– Run LPL Code , HTML Document –

Problem:

Listing 83: The Complete Model implemented in LPL [2]

```
model xDrawCircle "Drawing Circles";
  --Draw.Scale(3,3);
  Draw.Scale(2,-5);
  Draw.Circle(15,15,10,2,3,5);
  Draw.Circle('0.5',40,40,10,1,0);
  Draw.Ellipse('12.0',60,0,30,20,2,3,4,0.2,0.2);
  Draw.DefLine('dash',8,3,0);
  Draw.Ellipse('#dash','14.0',120,50,40,50,2,3,4);
  Draw.Triangle(0,80,20);
end
```

Output as follows:

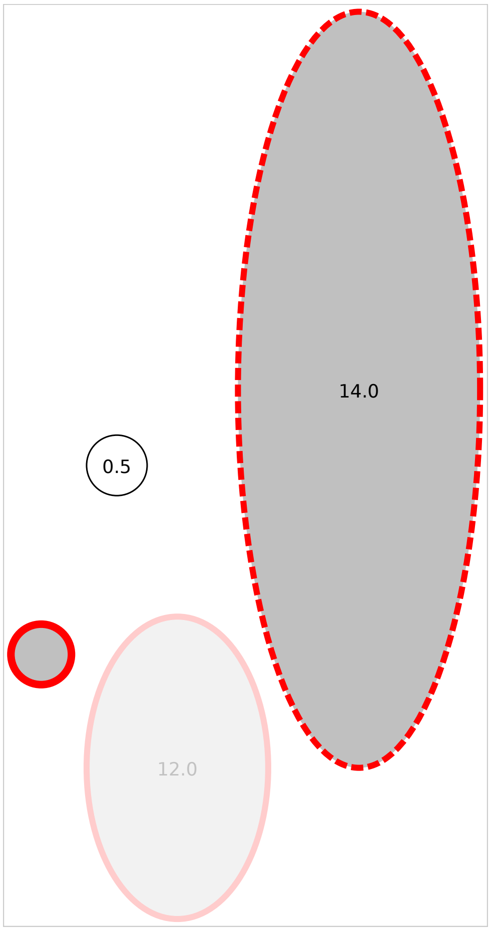


Figure 13: Output

83 A Quadratic Constraint (xDrawCircle1)

–– Run LPL Code , HTML Document –

Problem: This is a small model with a (convex) quadratic constraint. Note that this model can only be solved with particular solvers such as Cplex12. Therefore we force the use of this solver using the LPL function `SetSolver(...)`.

The submodel `draw` produces the picture in Figure 14.

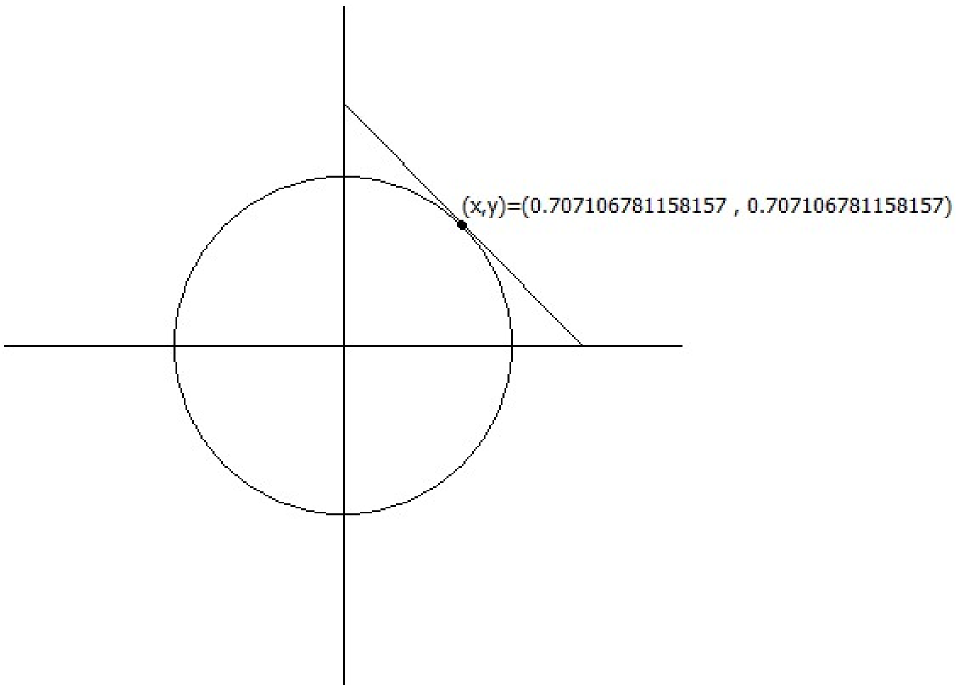


Figure 14: The Solution

The model is as follows:

Listing 84: The Complete Model implemented in LPL [2]

```
model xDrawCircle1 "A Quadratic Constraint";
  --SetSolver(knitroLSol);
  variable x; y; v; w;
  constraint A: x^2 + y^2 <= 1;
  maximize obj: x+y;
  Writep(x,y);
  draw;
  model draw;
    Draw.Scale(100,100);
    Draw.Circle(2,2,1,-1,0);
    Draw.Line(2,0,2,4);
    Draw.Line(0,2,4,2);
    Draw.Line(2,0.58,3.42,2,3,3);
    Draw.Circle(x+2,2-y,.04);
    Draw.Text('(x,y)␣=␣('&Round(x,-2)&'␣,␣'&Round(y,-2)&')',2+x,1.8-y);
  end
end
```

84 Show Color Numbers in LPL (xDrawColors)

–— Run LPL Code , HTML Document –

Problem: Colors assignment in LPL to positive integers.

Listing 85: The Complete Model implemented in LPL [2]

```
model xDrawColors "Show Color Numbers in LPL";
  set y:=[1..8]   "Eight color tables";
      x:=[1..32] "32 darkness in gradients";
  integer parameter a{y,x}:= 32*(y-1)+x-1;
  integer parameter b{y,x}:= 32*(y-1)+32-x;
  Draw.Scale(30,30);
  Draw.Line(1,1,2,1,1);
  Draw.Text('The␣color␣numbers␣0␣to␣255␣in␣LPL␣are␣as␣follows:',2,2,30)
      ;
  {y,x} Draw.Rect((32*(y-1)+x-1)&'',x,y+2,1,1,a);
  --- using RGB function for colors.
  {i in 1..256} Draw.Rect((i-1)/8,12,1/8,2,Rgb(0,0,i-1),Rgb(0,0,i-1));
  {i in 1..256} Draw.Rect((i-1)/8,15,1/8,2,Rgb(i-1,i-1,255),Rgb(i-1,i
      -1,255));
end
```

Output is as follows:

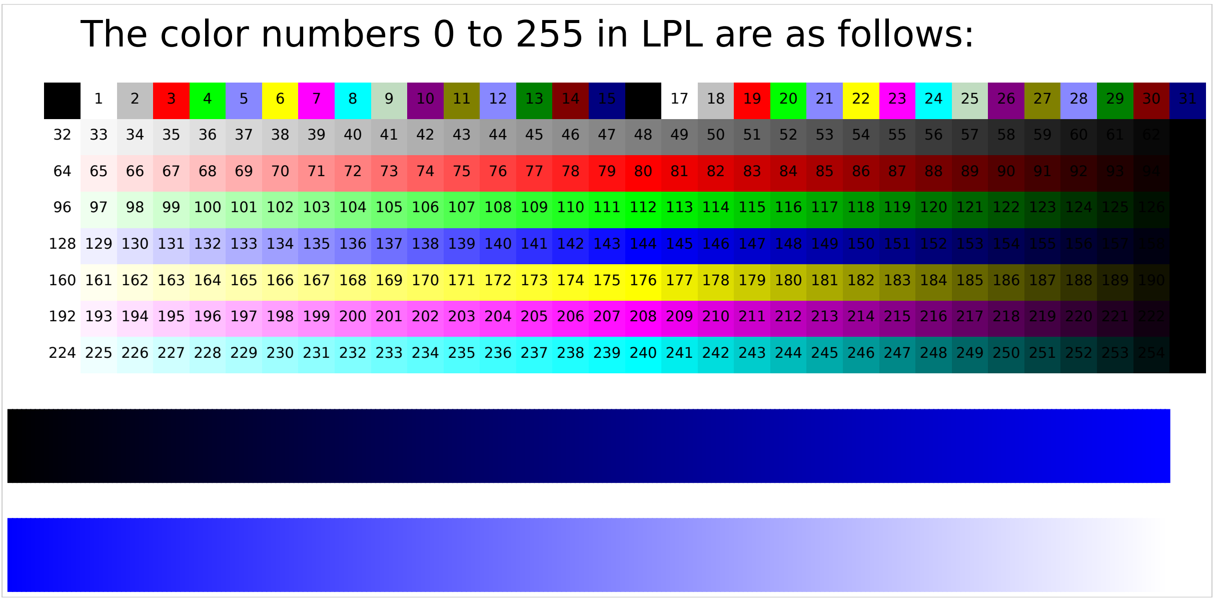


Figure 15: Output

85 Draw Filters (xDrawFilters)

—– Run LPL Code , HTML Document –

Problem: This model example shows SVG filters.

Listing 86: The Complete Model implemented in LPL [2]

```
model xDrawFilters "Draw Filters";
  Draw.DefFilter('f', 9,11,1, 4);            // 9=feGaussianBlur
  Draw.DefFilter('f',13,1 ,2, 4,4);          //13=feOffset
  Draw.DefFilter('f',14,1 ,3, 5,.75,20,-2,Rgb(187,187,187));
                                            //14=feSpecularLighting
  Draw.DefFilter('f',18 ,-5000,-10000,20000);   //18=fePointLight
  Draw.DefFilter('f', 4,3,11,3,2);              // 4=feComposite
  Draw.DefFilter('f', 4,10,3,4,6,0,1,1,0);      // 4=feComposite
  Draw.DefFilter('f',11);                       //11=feMerge
  Draw.DefFilter('f',24,2);                     //24=feMergeNode
  Draw.DefFilter('f',24,4);                     //24=feMergeNode
  Draw.Rect(1,20,198,85,Rgb(136,136,136),4);
  //Draw.Group('#f');
  Draw.Path('#f','M50,90␣C0,90␣0,30␣50,30␣L150,30␣C200,30␣200,90␣150,90
     ␣z',-1,5,10);
  Draw.Path('#f','M60,80␣C30,80␣30,40␣60,40␣L140,40␣C170,40␣170,80␣
     140,80␣z',5);
  //Draw.EndGroup();
  Draw.Text('#f','LPL',62,76,40);
  Draw.Rect(1,125,198,60,Rgb(136,136,136),4);
  Draw.Circle('#f',30,155,25,2);
  Draw.Rect('#f',70,130,50,50,3);
  Draw.Path('#f','M130,155␣l50,-25␣l0,50␣z',4);
end
```

Output as follows:

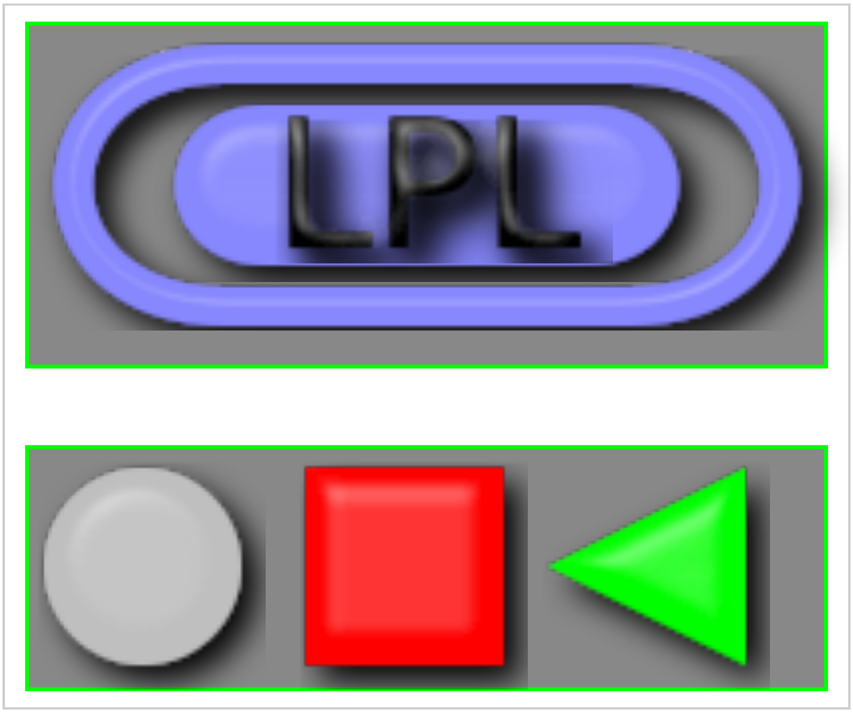


Figure 16: Output

86 Show radial gradient (xDrawGrad0)

–— Run LPL Code , HTML Document –

Problem:

Listing 87: The Complete Model implemented in LPL [2]

```
model xDrawGrad0 "Show radial gradient";
  Draw.DefFont('Verdana,sans-serif',50,-2,-2,2);
  parameter blue:=5; yellow:=6; red:=3;
  Draw.DefGrad('rgr1' ,1,.5,.5,.5, .5,.5, 0,blue,1,yellow);
  Draw.DefGrad('rgr2' ,1,.9,.5,.5, .9,.5, 0,blue,1,yellow);
  Draw.DefGrad('rgr3' ,1,.3,.7,.7, .3,.7, 0,blue,1,yellow);
  Draw.DefGrad('rgr4' ,1,.5,.5,.5, .5,.5, 0,blue,.5,yellow,1,blue);
  Draw.DefGrad('rgr5' ,1,.5,.5,.5,  1,.5, 0,blue,1,yellow);
  Draw.DefGrad('rgr6' ,1,.5,.5,.5, .5,1,  0,blue,1,yellow);
  Draw.DefGrad('rgr7' ,1,.8,.3,.9, .8,.3, 0,blue,1,yellow);
  Draw.DefGrad('rgr8' ,1,.5, 1,.9, .3,0,  0,blue,1,yellow);
  Draw.DefGrad('rgr9' ,1,.5,.5,.5, .5,.5, 0,blue,1,yellow);
  Draw.DefGrad('rgr10',2,.5,.5,.2, .5,.5, 0,blue,1,yellow); --
      spreadMethod='repeat',
  Draw.DefGrad('rgr11',3,.5,.5,.5, .5,.5, 0,blue,1,yellow); --
      spreadMethod='reflect
  Draw.DefGrad('rgr12',1,.5,.5,.5, .5,.5, .3,blue,.7,yellow);
  Draw.DefGrad('rgr_text',1,.5,.5,.8, .5,1, 0,red,.25,yellow,.5,blue
      ,.75,yellow,1,red);
  for{i in 1..12} do Draw.Rect('#rgr'&i,20+200*Trunc((i-1)/4),20+100*(i
      -1)%4,180,80); end
  --Draw.Text('#rgr_text','radialGradient&apos;s',50,465);
  Draw.Text('#rgr_text','radialGradient\'s',50,465);
end
```

Output as follows:

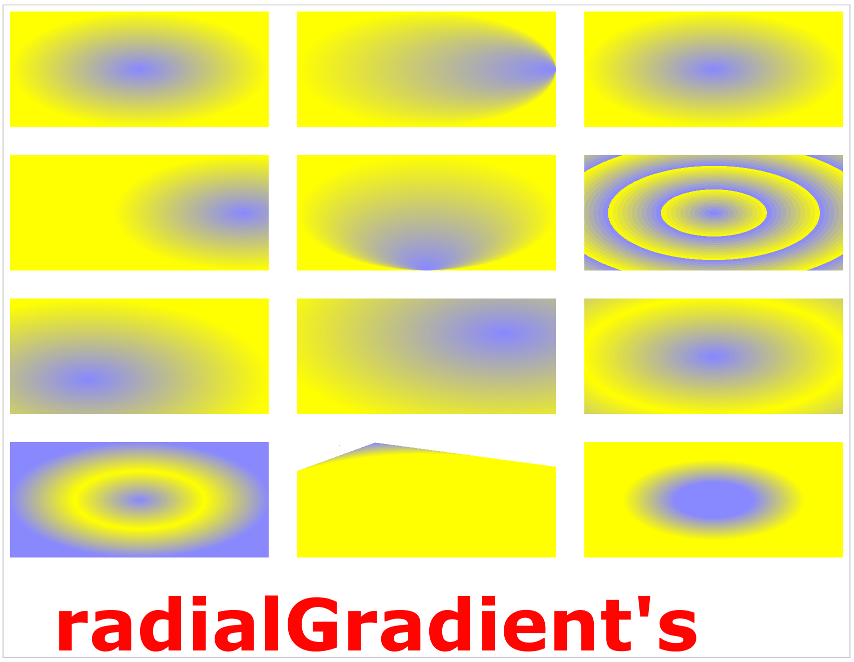


Figure 17: Output

87 Show linear gardient (xDrawGrad1)

—– Run LPL Code , HTML Document –

Problem:

Listing 88: The Complete Model implemented in LPL [2]

```
model xDrawGrad1 "Show linear gardient";
  Draw.DefFont('Verdana,sans-serif',50);
  Draw.DefFill('id',-2,0);
  parameter blue:=5; yellow:=6; red:=3; limegreen:=4; green:=13; white
      :=1;
  Draw.DefGrad('lgr1' ,1,0,0,1,0,0, 0,blue,1,yellow);
  Draw.DefGrad('lgr2' ,1,0,0,1,0,0, .2,blue,.8,yellow);
  Draw.DefGrad('lgr3' ,1,0,0,1,0,0, .4,blue,.6,yellow);
  Draw.DefGrad('lgr4' ,1,0,0,1,0,0, .5,blue,.5,yellow);
  Draw.DefGrad('lgr5' ,1,0,0,1,0,0, 0,red,1,limegreen);
  Draw.DefGrad('lgr6' ,1,0,0,0,1,0, 0,red,1,limegreen);
  Draw.DefGrad('lgr7' ,1,0,0,1,1,0, 0,red,1,limegreen);
  Draw.DefGrad('lgr8' ,1,0,1,1,0,0, 0,red,1,limegreen);
  Draw.DefGrad('lgr9' ,1,0,0,1,0,0, 0,blue,.25,yellow,.5,red,.75,green
      ,1,blue);
  Draw.DefGrad('lgr10',1,0,0,1,0,0, 0,blue,.2,yellow,.8,yellow,1,blue);
  Draw.DefGrad('lgr11',1,0,0,1,0,0, 0,blue,.1,yellow,.2,white,.9,blue);
  Draw.DefGrad('lgr12',1,0,0,1,1,0, 0,white,.3,yellow,.5,blue,.7,yellow
      ,1,white);
  for{i in 1..12} do Draw.Rect('#lgr'&i,20+200*Trunc((i-1)/4),20+100*(i
      -1)%4,180,80); end
  Draw.Text('#lgr9␣id','linearGradient\'s',50,465);
end
```

Output as follows:

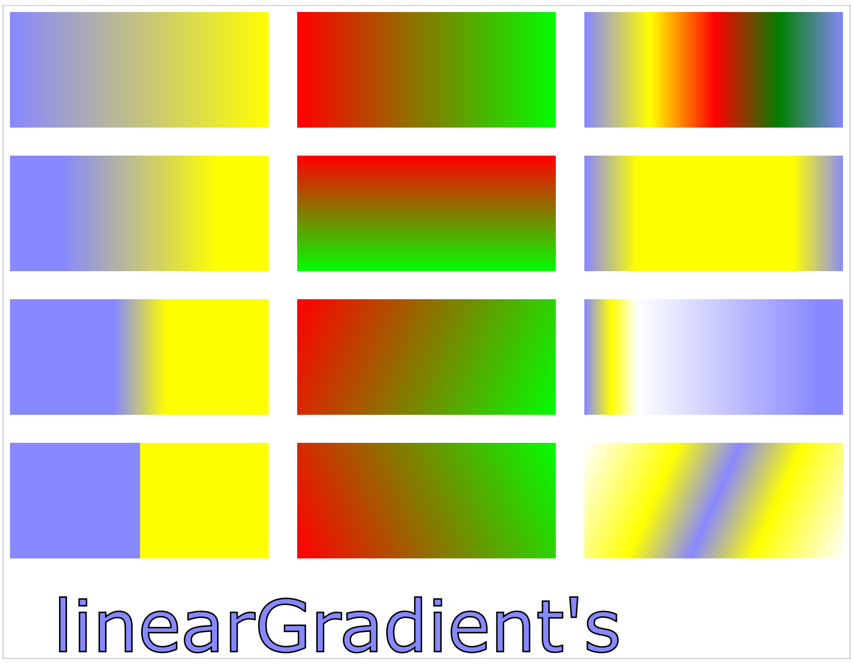


Figure 18: Output

88 Show figures with gradient (xDrawGrad2)

—- Run LPL Code , HTML Document –

Problem:

Listing 89: The Complete Model implemented in LPL [2]

```
model xDrawGrad2 "Show figures with gradient";
  // http://www.w3.org/Graphics/SVG/IG/resources/svgprimer.html#preface
  parameter white:=1; black:=0;
  Draw.DefGrad('rGr',-2,-2,-2,-2,-2,-2, 0,white,1,black);
  Draw.Path('M',100,200,200,200,150,100); Draw.Path('#rGr','Z',-2,0,2);
  Draw.DefGrad('lGr',-2,-2,-2,-2,-2,-2, 0,white,1,black);
  Draw.Path('M',210,200,310,200,260,100); Draw.Path('#lGr','Z',-2,0,2);
  Draw.DefGrad('rid',1,-2,-2,.65, .05,.05, 0,Rgb(0,238,0), 1,Rgb
      (0,102,0));
  Draw.Rect('#rid',340,100,100,100,-2,Rgb( 0,80,0),3,1,1,10,10);
end
```

Output as follows:

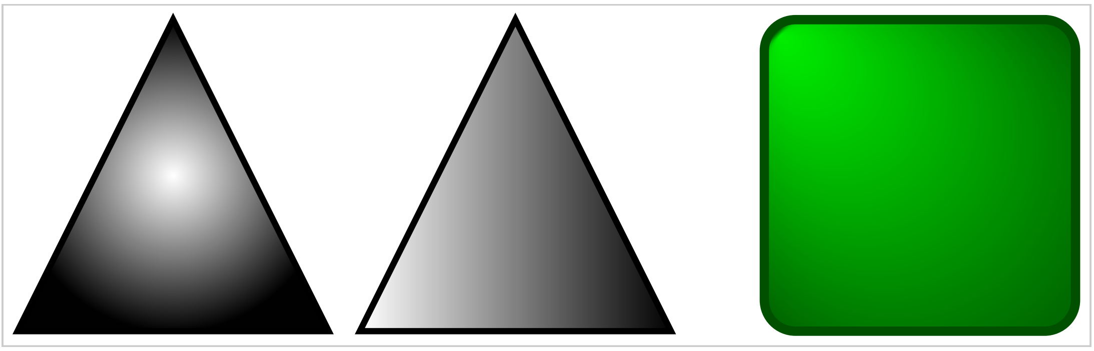


Figure 19: Output

89 Check Points inside a Polygon (xDrawInside)

—- Run LPL Code , HTML Document –

Problem: The Geom.Inside(X,Y,kxa,kya) checks whether a point (X,Y) is inside a (simple) polygon consisting of *k* points (xy,ya).

Listing 90: The Complete Model implemented in LPL [2]

```
model xDrawInside "Check Points inside a Polygon";
  set i:=1..1000  "generate 1000 points (X,Y)";
      k:=1..8;
  parameter PI:=3.14159;
     X{i}:=Rnd(0,1); Y{i}:=Rnd(0,1);
     xa{k}:=.5+Sqrt(1/5)*Sin(PI/#k+2*PI*(k-1)/#k);
     ya{k}:=.5+Sqrt(1/5)*Cos(PI/#k+2*PI*(k-1)/#k);
  Draw.Scale(100,100);
  // draw the polygon
  {k} Draw.Path('B',xa,ya); Draw.Path('z',1,0);
  // draw only the points not inside the polygon
  {i|~Geom.Inside(X,Y,{k}xa,{k}ya)} Draw.Circle(X,Y,.005);
end
```

Output as follows:

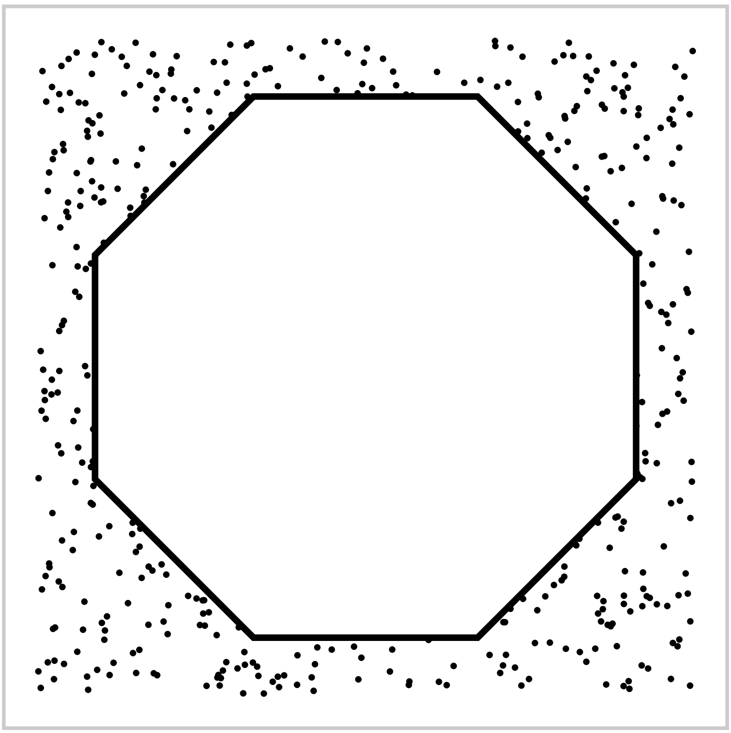


Figure 20: Output

90 Check Two Segments intersect (xDrawIntersect)

–– Run LPL Code , HTML Document –

Problem: The segment between the two points 8 and 4 and the segment between the two points 1 and 2 intersect, while The segment between the two points 5 and 2 and the segment between the two points 1 and 9 no *not* intersect.

Listing 91: The Complete Model implemented in LPL [2]

```
model xDrawIntersect "Check Two Segments intersect";
  set i:=1..10  "generate 100 points (X,Y)";
      k:=1..8;
  SetRandomSeed(1);
  parameter PI:=3.14159;
     X{i}:=Rnd(0,10); Y{i}:=Rnd(0,10);
     xa{k}:=.5+Sqrt(1/5)*Sin(PI/#k+2*PI*(k-1)/#k);
     ya{k}:=.5+Sqrt(1/5)*Cos(PI/#k+2*PI*(k-1)/#k);
  Draw.Scale(50,50);
  Draw.Line(X[8],Y[8],X[4],Y[4],0,3);
  Draw.Line(X[1],Y[1],X[2],Y[2],0,3);
  Draw.Line(X[5],Y[5],X[2],Y[2],3,3);
  Draw.Line(X[1],Y[1],X[9],Y[9],3,3);
  {i} Draw.Circle(i&'',X,Y,.2,1,0);
  parameter a:=Geom.Intersect(X,Y,8,4,1,2);
  parameter b:=Geom.Intersect(X,Y,5,2,1,9);
  Write('Line␣segment␣(8,4)␣does%s␣intersect␣with␣line␣segment␣(1,2)\n'
     , if(a,'','␣not'));
  Write('(Red␣line␣segment␣(5,2)␣does%s␣intersect␣with␣red␣line␣segment
     ␣(1,10)\n', if(b,'','␣not'));
end
```

The output as follows:

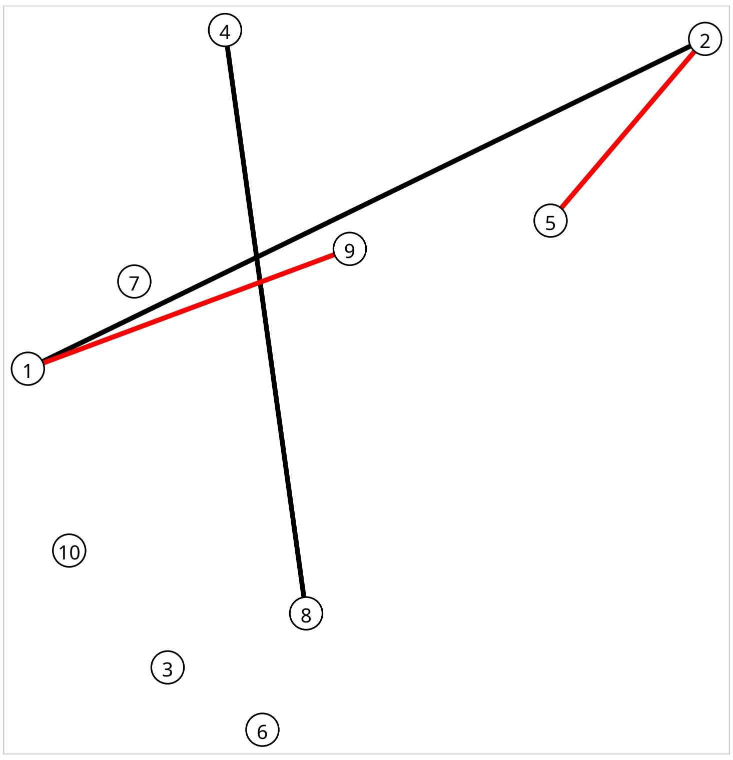


Figure 21: Output

91 Drawing Lines (xDrawLine)

–– Run LPL Code , HTML Document –

Problem:

Listing 92: The Complete Model implemented in LPL [2]

```
model xDrawLine "Drawing Lines";
  --Draw.Scale(3,3);
  --Draw.Scale(3,-3);
  Draw.Line('line',0,0,200,200);
  Draw.CLine('curve',0,20,180,200,-40);
/*  Draw.CArrow('another curve',0,40,160,200,-40,0,3);
  Draw.Line(20,0,200,180,4,5);
  Draw.Line('text follows line',40,0,200,160,4,5,-2,-.3);
  Draw.DefFont('curs','cursive',30,3,2);
  Draw.Line('#curs','red',60,0,200,140,2,2,-2,0.2);
  Draw.DefLine('dash',8,3,0);
  Draw.Line('#dash',80,0,200,120);
  Draw.Circle(0,0,2,5);
  Draw.Text('(0,0)',2,0);
  Draw.Circle(200,200,2);
*/
end
```

Output as follows:

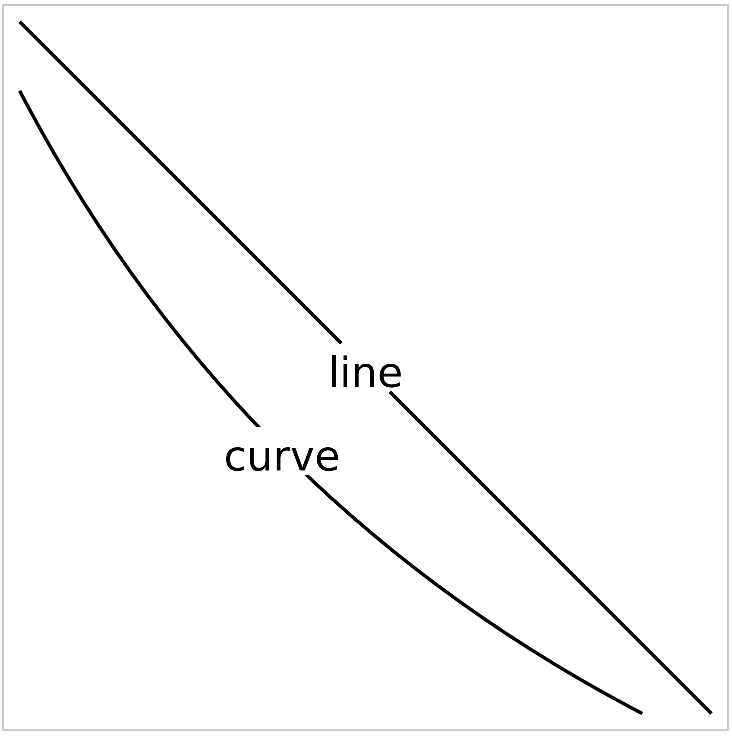


Figure 22: Output

92 Draw a path (xDrawPath)

–— Run LPL Code , HTML Document –

Problem:

Listing 93: The Complete Model implemented in LPL [2]

```
model xDrawPath "Draw a path";
  parameter white:=1; grey:=2;
  Draw.DefGrad('rGr',-2,-2,-2,-2,.7,.7, 0,white,1,grey);
  //Draw.DefFill('path',2,2);
  Draw.Rect(0,0,320,320,0,-1);
  Draw.Path('M',10,10);
  Draw.Path('V',20); Draw.Path('H',20);
  Draw.Path('C',20,20,100,10,230,230);
  Draw.Path('S',20,150,100,10);
  Draw.Path('#rGr','Z');
  Draw.Circle(240,280,5,1,0,3);
end
```

Output as follows:

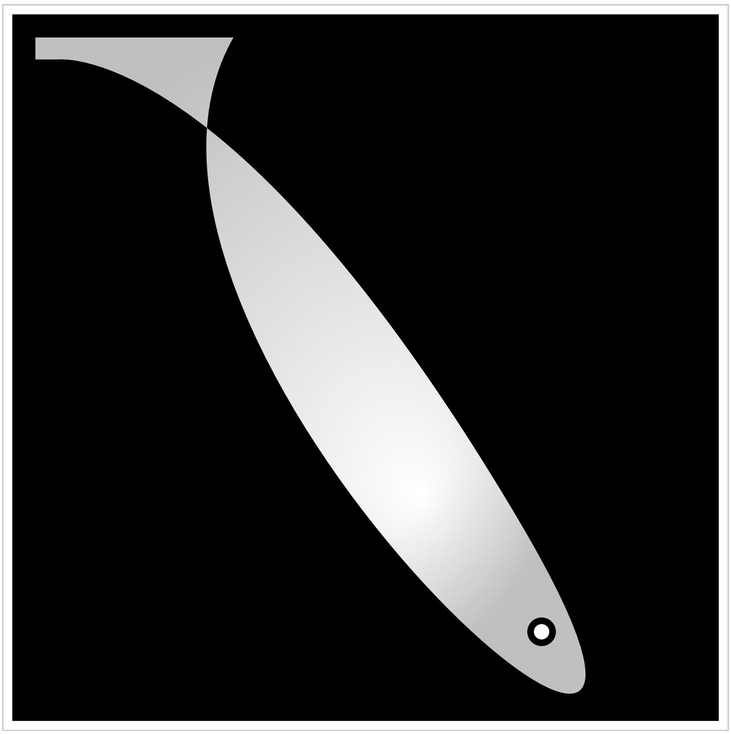


Figure 23: Output

93 Draw a pattern (xDrawPattern)

—- Run LPL Code , HTML Document –

Problem:

Listing 94: The Complete Model implemented in LPL [2]

```
model xDrawPattern "Draw a pattern";
  Draw.Scale(3,3);
  set i,j := [1..200];
  integer parameter c{i,j} := (i^2+2*j)%32+128;
  --integer parameter c{i,j} := ((i/10)^2+2*j)%32+64;
  --integer parameter c{i,j} := (Sqrt(10*i*j))%32+128;
  {i,j} Draw.Rect(i,j,1,1,c);
end
```

Output as follows:

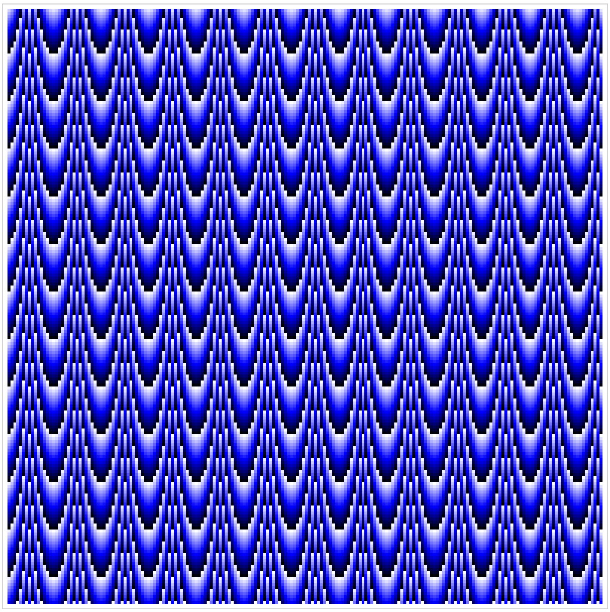


Figure 24: Output

94 Load Picture from File (xDrawPict)

—- Run LPL Code , HTML Document –

Problem:

Listing 95: The Complete Model implemented in LPL [2]

```
model xDrawPict "Load Picture from File";
  Draw.Scale(1,1,0,0,700,500);
  Draw.Picture('xDrawPict.jpg');
end
```

Output as follows:

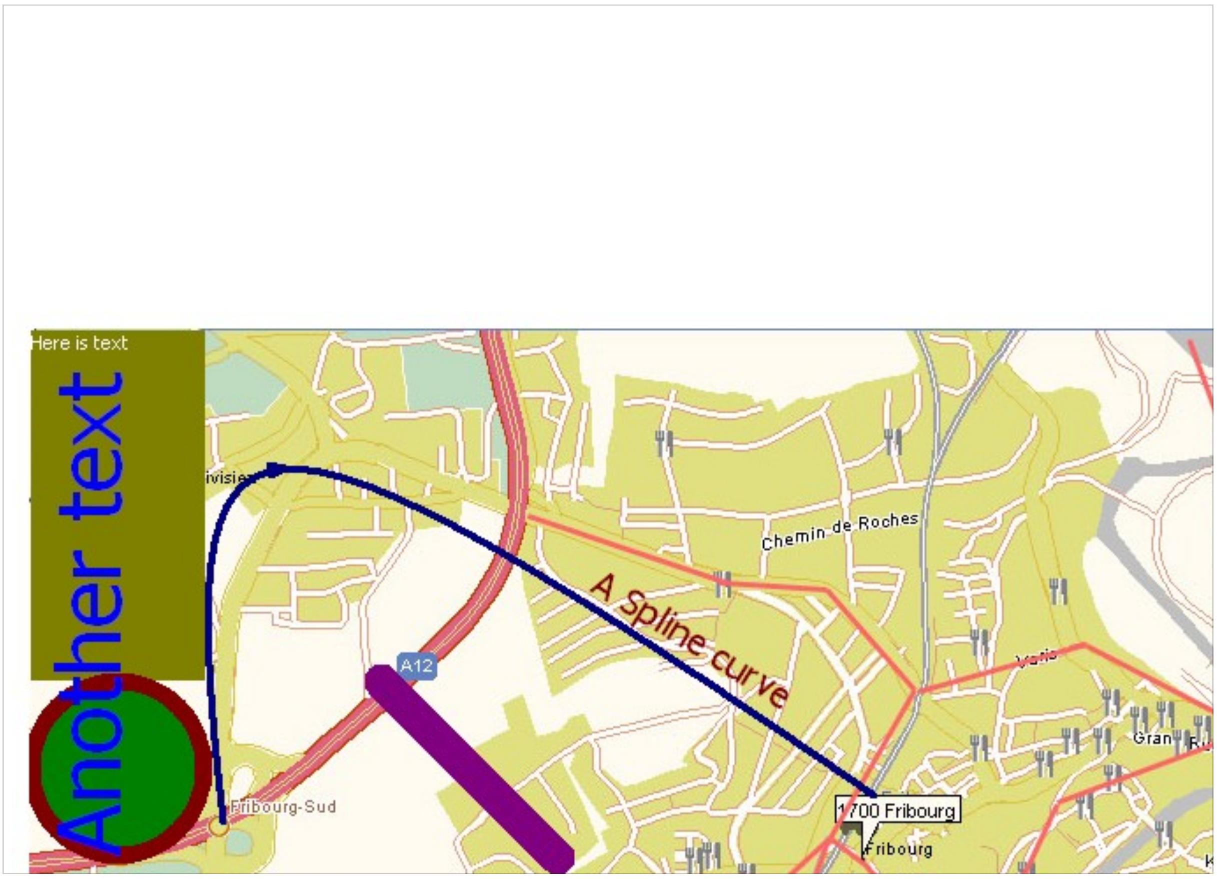


Figure 25: Output

95 Some drawing functions (xDrawPict1)

–– Run LPL Code , HTML Document –

Problem:

Listing 96: The Complete Model implemented in LPL [2]

```
model xDrawPict1 "Some drawing functions";
  Draw.Picture('xDrawPict1.jpg',0,0,700,500); --load and draw a
     background
  Draw.Rect(1,1,100,200,3);          --draw a rectangle
  Draw.Line(200,200,300,300,5,20); --draw an line
  Draw.Ellipse(100,200,100,150,7,10,10); --draw an ellipse
  Draw.DefFont('Tahoma',25);         --sets the font and its height
  Draw.Text('Here␣is␣text',1,1,12); --write a text to the picture
  Draw.Text('Another␣text',1,300,14,60,90);        --write a text to
     the picture
  Draw.Text('A␣Spline␣curve',330,130,12,25,-32);  --write a text to the
      picture
  Draw.CArrow(110,280,480,265,400); --draw a spline curve
  --Draw.Save('save.jpg');        --save the picture as jpg
end
```

Output as follows:

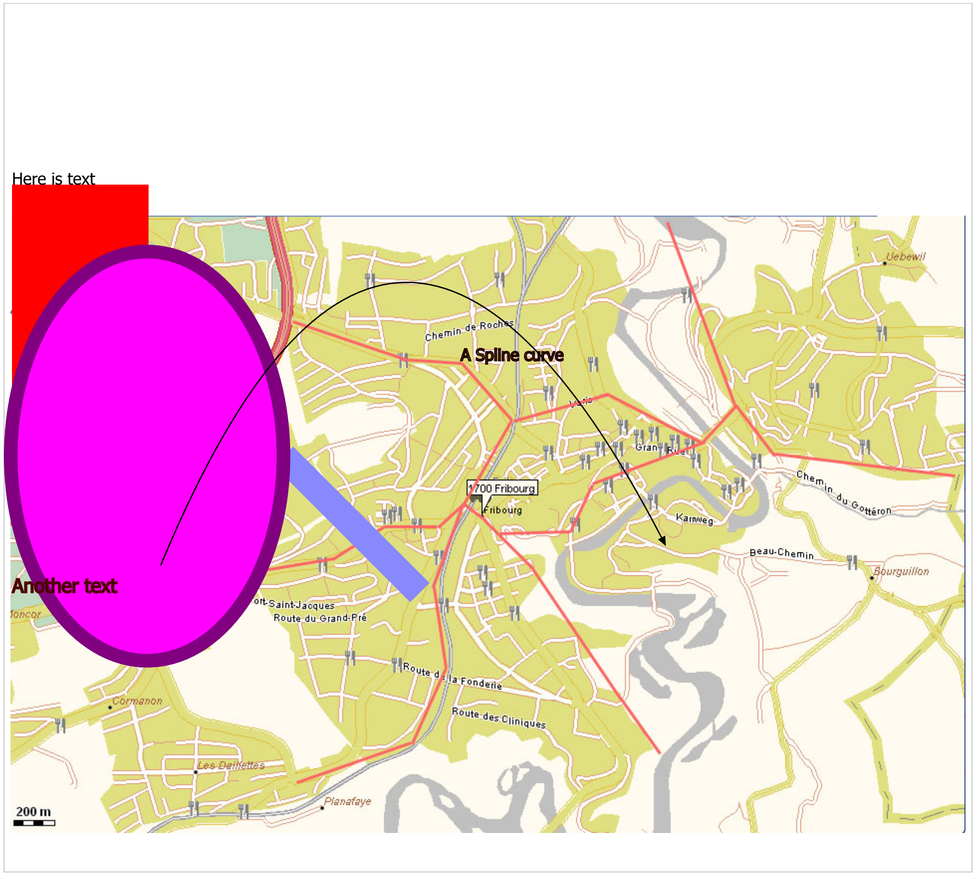


Figure 26: Output

96 Puzzle 64=65 (xDrawPuzzle)

–– Run LPL Code , HTML Document –

Problem: Look at the graph in Figure 27. Can you see the error?

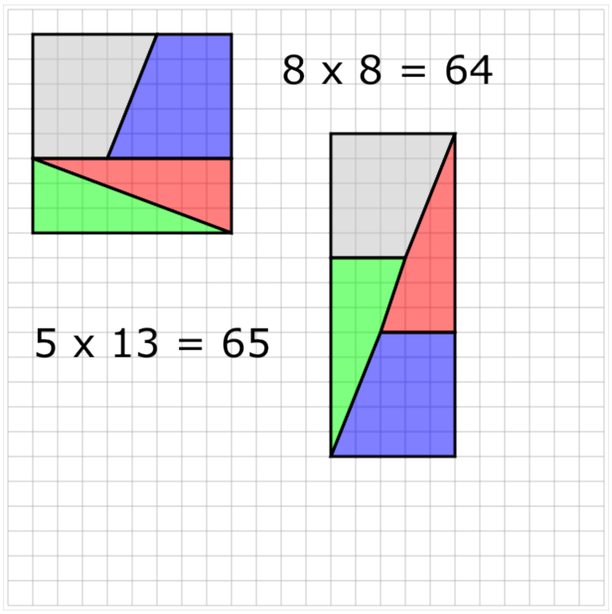


Figure 27: $64 \overset{?}{=} 65$

Modeling Steps

It is very easy to see that the red and green "triangles" on the right figure are not really triangles (if you are not convinced then set `n:=5` in the model and run again). A quick calculation also shows that they *cannot* be triangles. The proportion of the height and the length of these triangles is $3/8$. At the position 5 at the length, the height of this "triangle" is 2. But we know that $3/8 \neq 2/5$. Hence, these forms are *not* triangles.

These two graphs are intrinsicly linked to the Fibonacci numbers, for example, within the graph the three numbers 5, 8, and 13 are the consecutive Fibonacci numbers F_5, F_6, and F_7. These numbers are defined as follows:

$$F_1 = 1\ , \quad F_2 = 1\ , \quad F_n = F_{n-1} + F_{n-2}\ , \quad \text{with } n \geq 2$$

The Fibonacci numbers have many interesting properties, one of them is

$$F_{n+1} \cdot F_{n-1} - F_n^2 = (-1)^n\ , \quad \text{with } n \geq 1$$

Proof:

1. The property is true for $n = 1$, since $1 \cdot 0 - 1^2 = (-1)^1$.

2. Supposing the property is valid for n, then it is valid for $n + 1$, namely we substitute F_{n-1} with $F_{n+1} - F_n$ in the property and we get:

$$F_{n+1}^2 - F_{n+1}F_n - F_n^2 = (-1)^n$$

By multiplying with -1 and transforming it we finally get:

$$F_{n+2} \cdot F_n - F_{n+1}^2 = (-1)^{n+1}$$

That proves the property.

3. In particular, for $n = 6$ we have: $13 \cdot 5 - 8^2 = (-1)^6$ (see Figure 27).

An LPL model that draws this puzzle is given below. Open it and try different values for `n` (line 4 in the code).

Listing 97: The Complete Model implemented in LPL [2]

```
model xDrawPuzzle "Puzzle 64=65";
  set k:=1..20;
  parameter F{k}:=if(k=1,1,k=2,1,F[k-1]+F[k-2]); //Fibonacci numbers
    n:=6; -- for different sizes : try n:=5,6,7 and 8
    x:=2; y:=2;
  set i:=1..F[n+1]+10;
  Draw.Scale(25,25);
  Draw.DefFont('Verdana',40);
  {i}Draw.Line(1,i,#i,i,2);
  {i}Draw.Line(i,1,i,#i,2);
  Draw.Path('M',x,y,x+F[n-1],y,x+F[n-2],y+F[n-1],x,y+F[n-1]);
    Draw.Path('Z',2,0,3,.5);
  Draw.Path('M',x+F[n-1],y,x+F[n],y,x+F[n],y+F[n-1],x+F[n-2],y+F[n-1]);
    Draw.Path('Z',5,0,3,.5);
  Draw.Path('M',x,y+F[n-1],x+F[n],y+F[n-1],x+F[n],y+F[n]);
    Draw.Path('Z',3,0,3,.5);
  Draw.Path('M',x,y+F[n-1],x+F[n],y+F[n],x,y+F[n]);
    Draw.Path('Z',4,0,3,.5);
  Draw.Text(F[n]&'␣x␣'&F[n]&'␣=␣'&F[n]^2,x+F[n]+2,4);
  x:=x+F[n]+4; y:=y+4;
  Draw.Path('M',x,y,x+F[n-1],y,x+F[n-2],y+F[n-1],x,y+F[n-1]);
    Draw.Path('Z',2,0,3,.5);
  Draw.Path('M',x+F[n-3],y+F[n],x+F[n-1],y+F[n],x+F[n-1],y+F[n+1],x,y+F
     [n+1]);
    Draw.Path('Z',5,0,3,.5);
  Draw.Path('M',x+F[n-1],y,x+F[n-1],y+F[n],x+F[n-3],y+F[n],x+F[n-2],y+F
     [n-1]);
    Draw.Path('Z',3,0,3,.5);
  Draw.Path('M',x,y+F[n-1],x+F[n-2],y+F[n-1],x+F[n-3],y+F[n],x,y+F[n
     +1]);
    Draw.Path('Z',4,0,3,.5);
  Draw.Text(F[n-1]&'␣x␣'&F[n+1]&'␣=␣'&F[n-1]*F[n+1],2,2+F[n+1]);
end
```

This puzzle shows an important aspect in the modeling process: How can we discover generalizations of a result! Try to construct the puzzle for $n = 7$: we get

$$21 \cdot 8 - 13^2 = (-1)^7 \quad (168 = 169 - 1)$$

The corresponding figure can be constructed and is shown in Figure 28.

In the puzzle with $n = 5$ (Figure 29), we see immediately, that the two forms are not congruent.

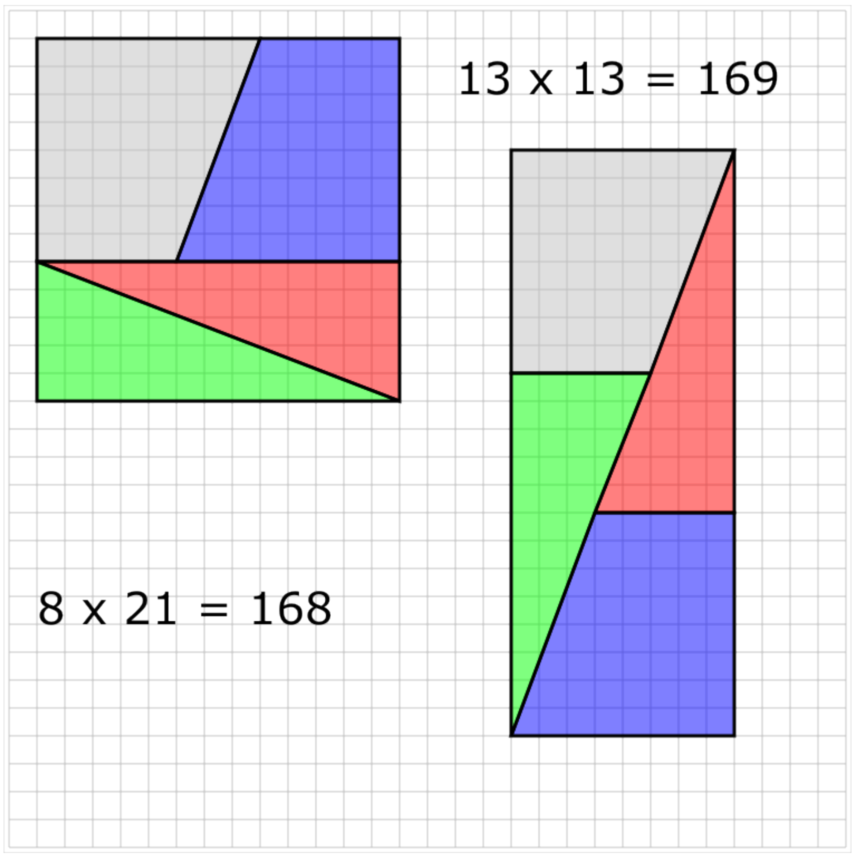


Figure 28: $169 \overset{?}{=} 168$

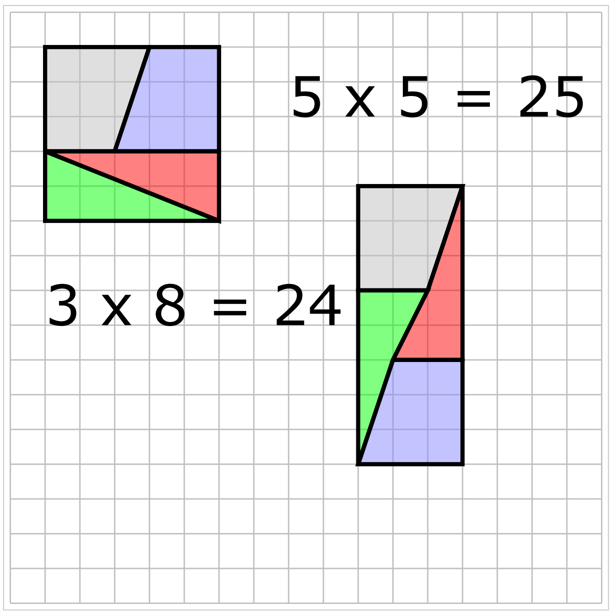


Figure 29: $25 \overset{?}{=} 24$

97 Draw rectangles (xDrawRect)

—- Run LPL Code , HTML Document –

Problem:

Listing 98: The Complete Model implemented in LPL [2]

```
model xDrawRect "Draw rectangles";
  --Draw.Scale(3,3);
  Draw.Scale(3,-3);
  Draw.Rect('a',100,100,30,40,2,3,5);
  Draw.Rect('rect',100,70,20,20,1,0);
  Draw.Rect('12.0',60,0,30,20,2,3,4,0.2,0.2);
  Draw.Rect('14.0',95,0,40,40,2,3,4,-2,-2,10,10);
  Draw.Rect(0,0,150,150,-1,3,2); //the large rect
  parameter Pi:=3.14159; x1:=100; y1:=45;
    an:=Arctan(y1/x1)*180/Pi;
    r:=Sqrt(x1^2+y1^2);
    x2:=r*Cos((an+50)*Pi/180); y2:=r*Sin((an+50)*Pi/180);
  Draw.Text('(x1,y1)',x1+3,y1+2);
  Draw.Text('(x2,y2)',x2-21,y2-3);
  Draw.DefTrans('tr','t',x1,y1);
  Draw.DefTrans('ro','r',50);
  Draw.Rect('#ro␣tr','hu',0,0,20,30,1,0);
  Draw.Arc('&#945;=50&#186;',0,0,r,an,an+50,3,10);
  Draw.Circle(x1,y1,2);
  Draw.Line(0,0,x1,y1);
  Draw.Circle(x2,y2,2);
  Draw.Line(0,0,x2,y2);
  parameter x:=50; y:=60;
  Draw.DefTrans('ro1','r',50,x,y);
  Draw.Rect('#ro1','hi',x,y,20,30,-1,0);
  Draw.Circle(x,y,2);
  Draw.Text('(x,y)',x+2,y-2);
end
```

The output as follows:

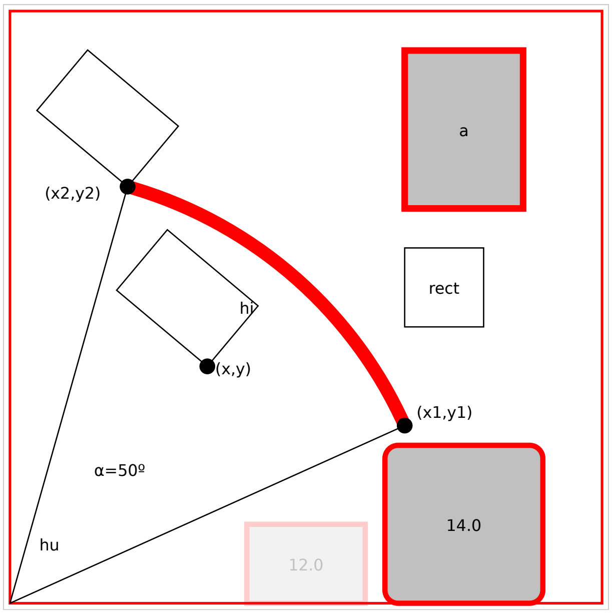


Figure 30: Output

98 Draw.Scale Function (xDrawScale)

—- Run LPL Code , HTML Document –

Problem: The first Scale instruction (outcommanded) scales the graph automatically, the second does it manually: the size of the graphic is 240 × 240 and the left/top of the graphic is (-120,-120) and (2,-2) are the scaling factors. If the y-scaling factor is negative then the graphic drawn upside down.

Listing 99: The Complete Model implemented in LPL [2]

```
model xDrawScale "Draw.Scale Function";
  --Draw.Scale(2,2);
  Draw.Scale(2,-2,-120,-120,240,240);
  Draw.Rect(-100,-100,200,200);
  Draw.Circle(-100,-100,2,3);
  Draw.Circle(100,100,2,4);
  Draw.Circle(-100,100,2,3);
  Draw.Circle(100,-100,2,4);
  Draw.Text('(-100,-100)',-115,-110);
  Draw.Text('(100,100)',85,110);
end
```

Output as follows:

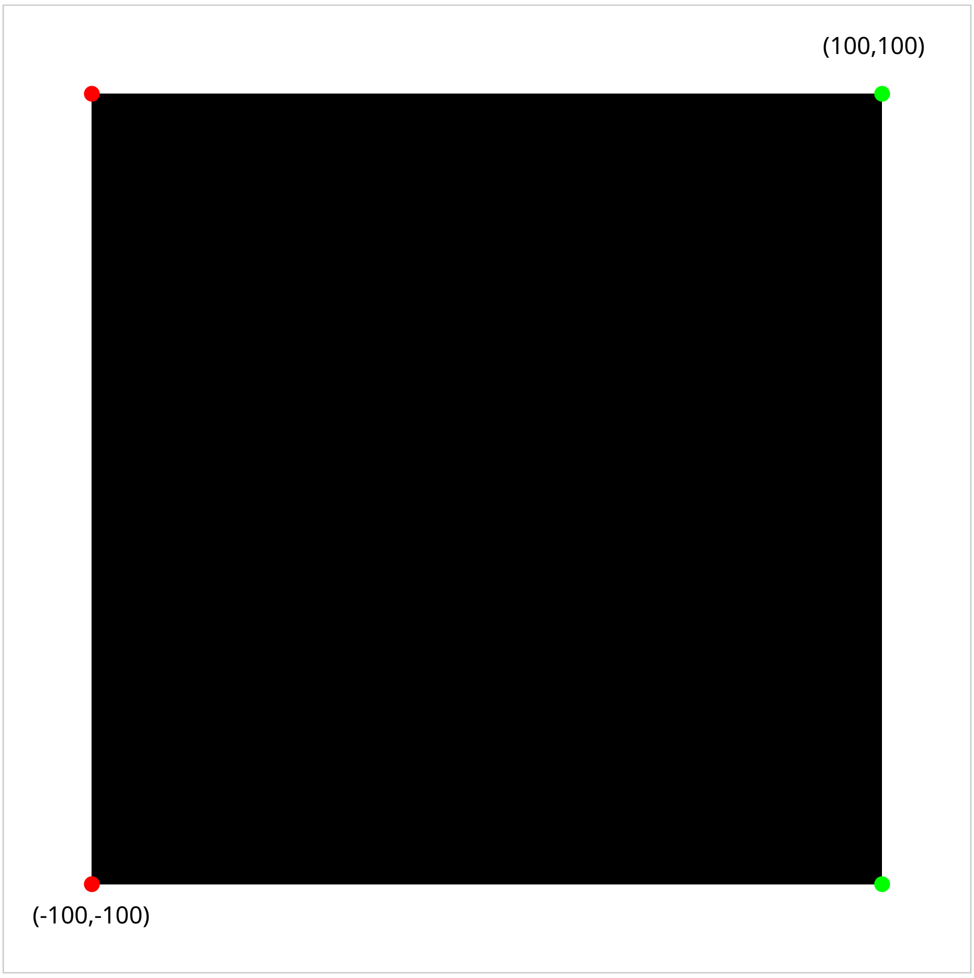


Figure 31: Output

99 Drawing Text (xDrawText)

–– Run LPL Code , HTML Document –

Problem: Text fonts in different shapes and sizes.

Listing 100: The Complete Model implemented in LPL [2]

```
model xDrawText "Drawing Text";
  --Draw.Scale(3,3);
  Draw.Scale(3,3,0,-10,200,150);
  parameter FontSize:=30;
  --Draw.DefFont('serif',FontSize); //default
  Draw.DefFont('seri','serif',FontSize);
  Draw.DefFont('sans','sans-serif',FontSize);
  Draw.DefFont('curs','cursive',FontSize);
  Draw.DefFont('fant','fantasy',FontSize);
  Draw.DefFont('mono','monospace',FontSize);
  Draw.Text('Hello␣World!␣default',20,0);
  Draw.Text('#seri','Hello␣World!␣serif',20,20);
  Draw.Text('#sans','Hello␣World!␣sans-serif',20,40);
  Draw.Text('#curs','Hello␣World!␣cursive',20,60);
  Draw.Text('#fant','Hello␣World!␣fantasy',20,80);
  Draw.Text('#mono','Hello␣World!␣monospace',20,100);
  Draw.DefFill('fill',3,4,3,0.2,0.5);
  Draw.Text('#fant␣fill','Hello␣World!!!',20,130,100);
  Draw.Circle(0,0,1);
  Draw.Text('(0,0)',2,0);
end
```

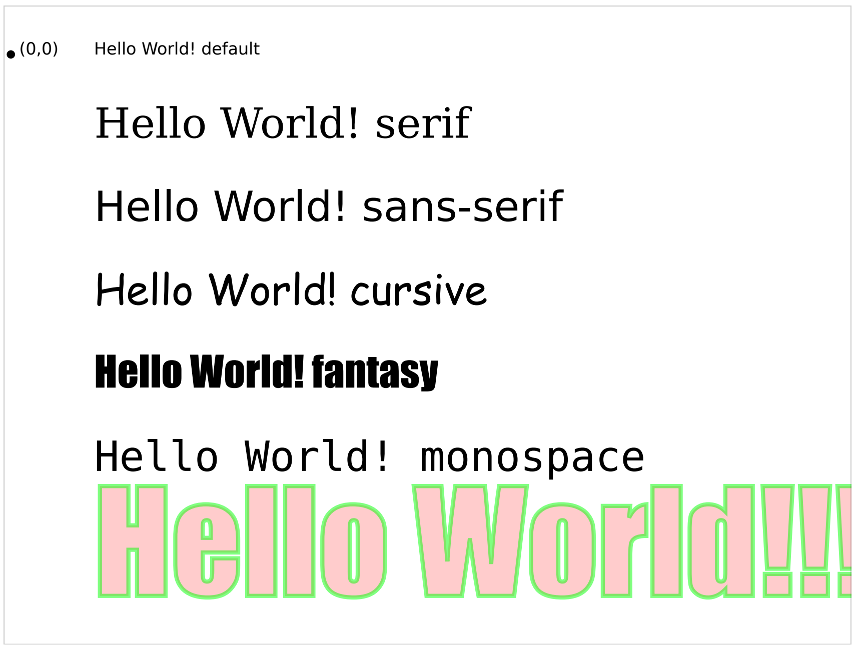


Figure 32: Output

100 Drawing a TextPath (xDrawTextPath)

—– Run LPL Code , HTML Document –

Problem:

Listing 101: The Complete Model implemented in LPL [2]

```
model xDrawTextPath "Drawing a TextPath";
  Draw.Scale(2,2,0,0,150,210);
  -- this defines a single path with id 'aa'
  Draw.Path('aa','M',50,50);
  Draw.Path('aa','A',30,30,0,0,1,130,50);
  Draw.Path('aa','L',130,200);
  -- this uses the path for the text
  Draw.Text('#aa','Text␣on␣a␣round␣corner,␣and␣more␣text.␣hello!'
      ,0,0,28);
end
```

Output as follows:

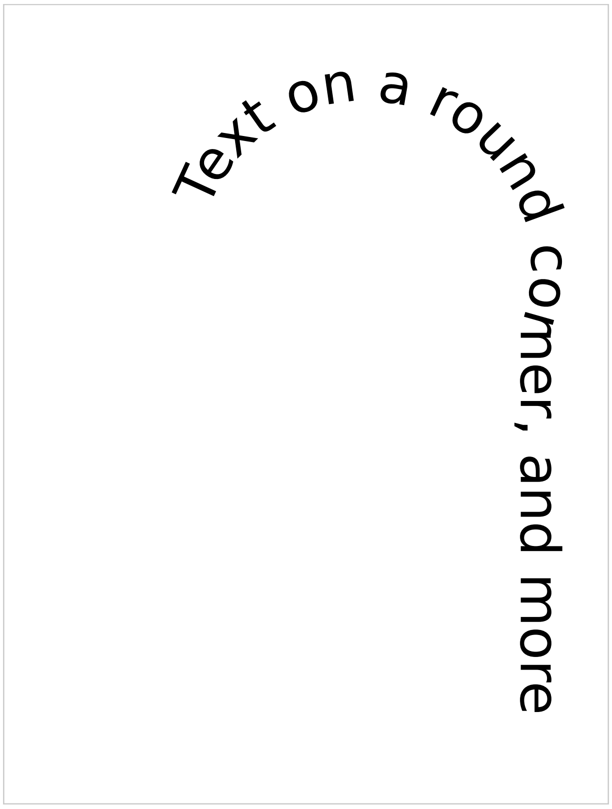


Figure 33: Output

101 Draw a Text on a Path (xDrawTPath)

–– Run LPL Code , HTML Document –

Problem:

Listing 102: The Complete Model implemented in LPL [2]

```
model xDrawTPath "Draw a Text on a Path";
  Draw.Scale(1,1,-5,145,360,210);
  Draw.Rect(0,150,350,200,1,0);
  Draw.Path('mytext','M␣20,255␣L␣100,255␣A␣70␣100␣0␣0␣0␣180,255␣A␣70␣
      100␣0␣0␣1␣260,255␣L␣370,255␣y',-1,3,1);
  Draw.Text('#mytext','this␣is␣a␣longer␣text␣on␣a␣path␣with␣downs␣and␣
      ups',100,100);
end
```

Output as follows:

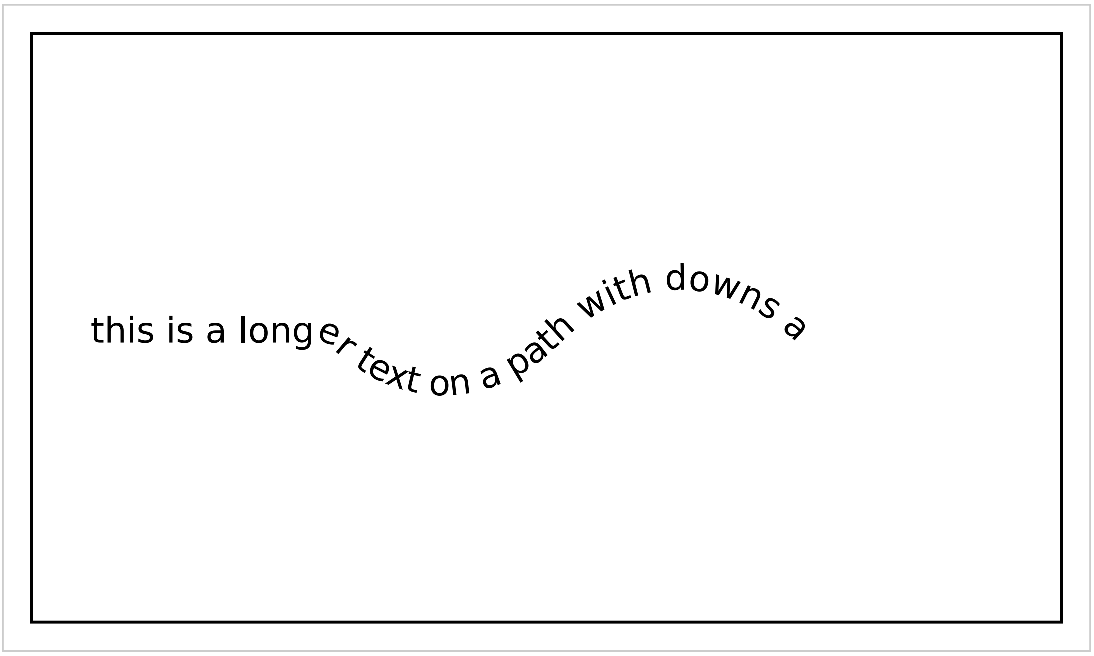


Figure 34: Output

102 Show transformations (xDrawTrans1)

—- Run LPL Code , HTML Document –

Problem:

Listing 103: The Complete Model implemented in LPL [2]

```
model xDrawTrans1 "Show transformations";
  Draw.Scale(2,2,0,0,130,100);
  --Draw.Scale(2,2); //does not a good job!
  for{i in 1..10} do
    Draw.DefTrans('id'&i,'t',10+3*i,10);
    Draw.DefTrans('id'&i,'r',10*i,40,40);
    Draw.Rect('#id'&i,10,10,50,50,i+3,i+2,3,2,2,.1,.1);
  end;
  --Draw.DefTrans('id1','r',45,130,30);
  --Draw.Rect('#id1',120,20,50,50);
  --Draw.Rect(100,100,90,90);
end
```

The output is as follows:

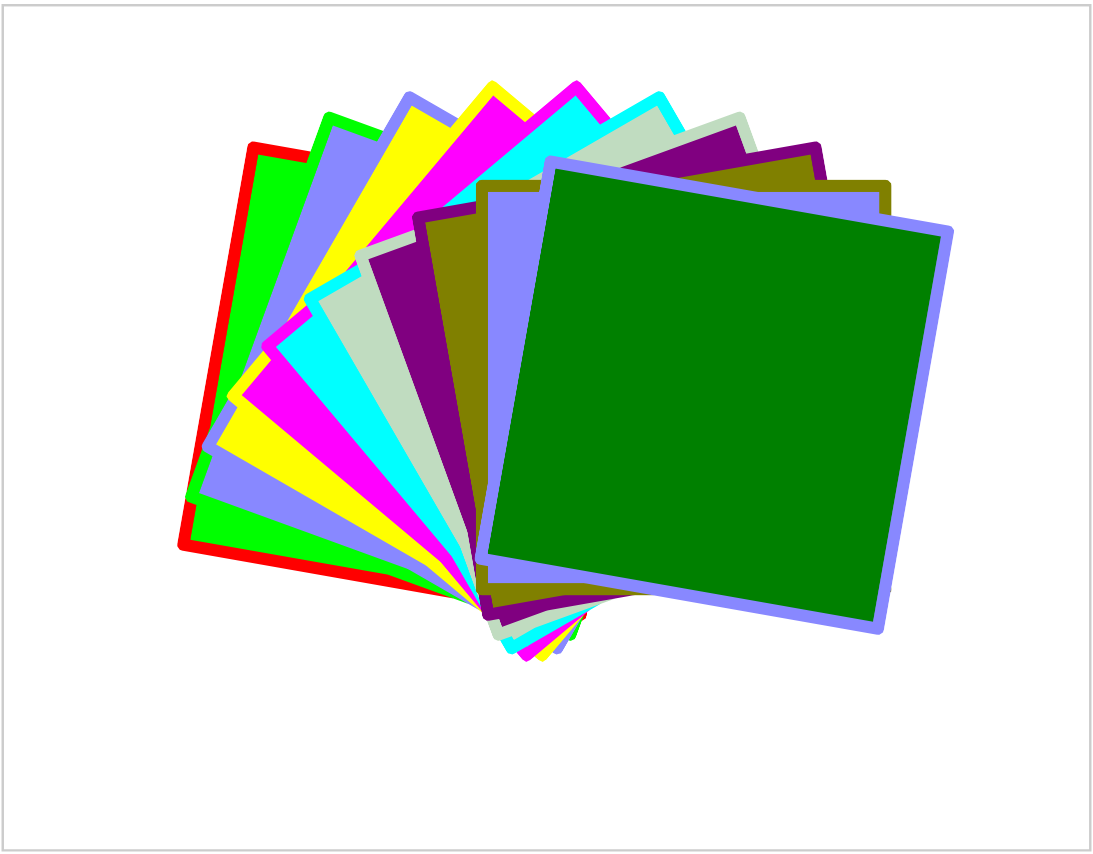


Figure 35: Output

103 Draw Penrose Triangle (xDrawxPenrose)

—– Run LPL Code , HTML Document –

Problem:

Listing 104: The Complete Model implemented in LPL [2]

```
model xDrawPenrose "Draw Penrose Triangle";
  parameter a:=300; b:=40; c:=Sqrt(1/3)*b;
    d:= a-3*c; e:=Sqrt(3/4)*(d+c)-2*b; f:=1/2*a-3*c;
    bColor:=2; fColor:=Rgb(2,30,40); tColor:=1;
  Draw.Scale(1,1,0,10,500,500);
  Draw.Rect(0,0,700,700,bColor);
  Draw.DefGrad('nr1',3,0,0,1,1,-2,0,fColor,1,tColor);
  Draw.DefTrans('trans1','r',120,100+f+2*c,400-e-4*b);
  Draw.DefTrans('trans2','r',240,100+a+c,400-b);
  Draw.Path('M',100,400); Draw.Path('l',a,0); Draw.Path('l',c,-b);
    Draw.Path('l',-c-d,0); Draw.Path('l',f,-e); Draw.Path('l',-c,-b);
    Draw.Path('#nr1','Z');
  Draw.Path('M',100+f+2*c,400-e-4*b); Draw.Path('l',a,0); Draw.Path('l'
     ,c,-b);
    Draw.Path('l',-c-d,0); Draw.Path('l',f,-e); Draw.Path('l',-c,-b);
    Draw.Path('#nr1␣trans1','Z');
  Draw.Path('M',100+a+c,400-b); Draw.Path('l',a,0); Draw.Path('l',c,-b)
     ;
    Draw.Path('l',-c-d,0); Draw.Path('l',f,-e); Draw.Path('l',-c,-b);
    Draw.Path('#nr1␣trans2','Z');
end
```

The output is as follows:

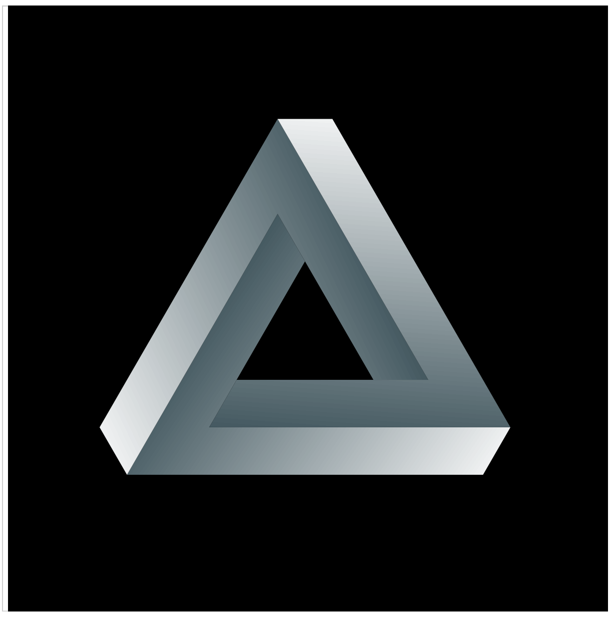


Figure 36: Penrose Triangle

104 Draw Sierpinski Triangles (xDrawxSierpinski)

—– Run LPL Code , HTML Document –

Problem:

Listing 105: The Complete Model implemented in LPL [2]

```
model xDrawSierpinski "Draw Sierpinski Triangles";
  parameter s:=400; t:= 5; c; f:=3^Ceil(Log(s/t)/Log(2));
  set i,j:=1..f;
  parameter x{i}; y{i}; xx{j}; yy{j};;
  x[1] := 1; y[1] := 1;
  while s>t do
    s := s/2; c := 1;
    {i|x}(xx[c]:=x, yy[c]:=y, xx[c+1]:=x+s, yy[c+1]:=y,
          xx[c+2]:=x+s/2, yy[c+2]:=y+Sqrt(3/4*s^2), c:=c+3);
    {j}(x[j]:=xx, y[j]:=yy);
  end
  Draw.DefFill('pa',1,0,-2,0.5);
  {i|x}(Draw.Path('M',x,y,x+s,y,x+s/2,y+Sqrt(3/4*s^2),x,y),Draw.Path('#
     pa','Z'));
end
```

The output is as follows:

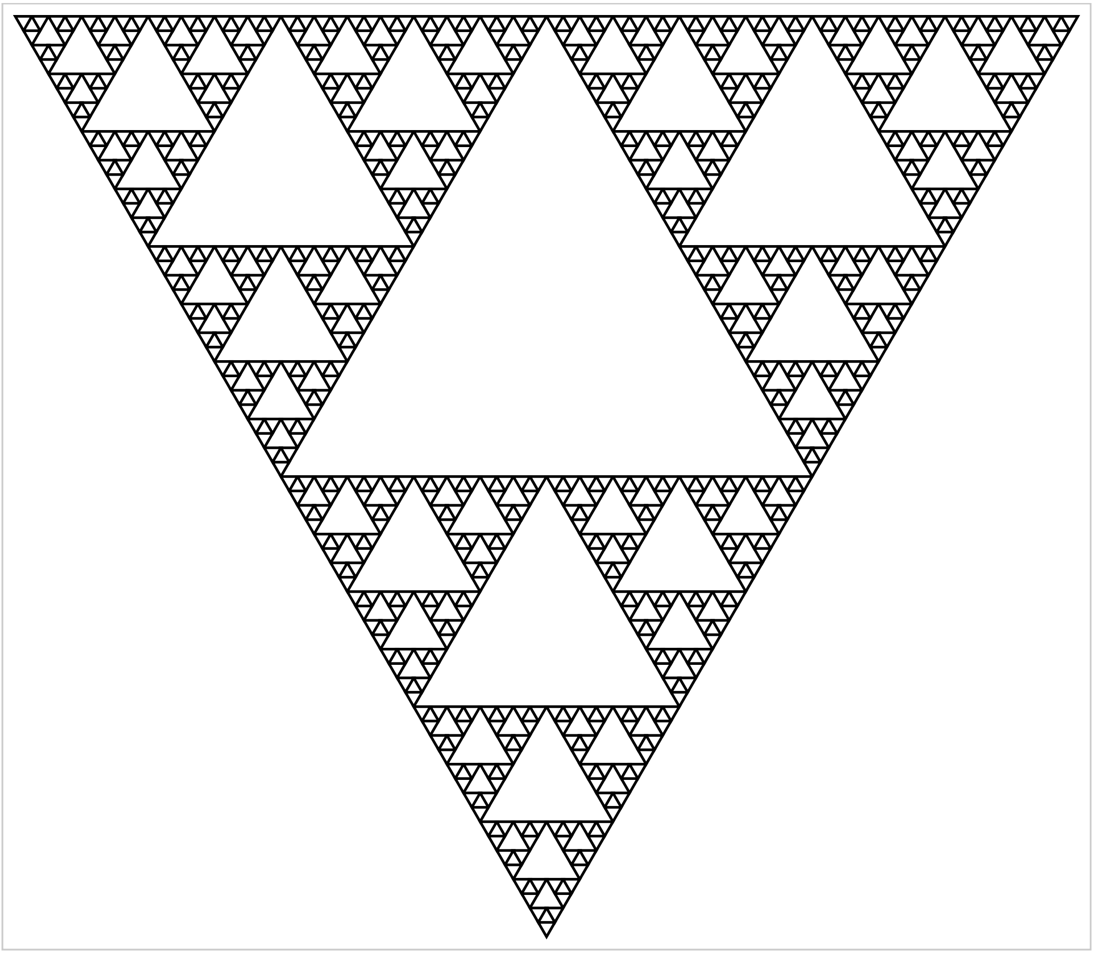


Figure 37: Sierpinski Triangle

105 Draw a colored spiral (xDrawxSpiral)

—– Run LPL Code , HTML Document –

Problem: Draw a rectagle filled with a linear gradient from `fromColor` to `toColor` and from top left $(0,0)$ to bottom right $(1,1)$. Then repeat this step by slidely rotating and decreasing the size of the rectangle.

Listing 106: The Complete Model implemented in LPL [2]

```
model xDrawSpiral " Draw a colored spiral";
  set r := [1..100]  "repaet";
  parameter size:=400; a:=4; rad:=a/180*3.14159265; c; d; s;
  parameter fromColor:=2; toColor:=5;
  Draw.DefGrad('grad0',3, 0,0,1,1, -2, 0,fromColor,1,toColor);
  Draw.Rect('#grad0',0,0,size,size);
  s:=size;
  {r}(Draw.DefTrans('rot','r',a,size/2,size/2),
      Draw.DefGrad('grad'&r,3, 0,0,1,1, -2, 0,fromColor,1,toColor),
      c:=Arctan(rad)*s/(1+Arctan(rad)), d:= s-c, s:=Sqrt(c^2+d^2),
      Draw.Rect('#rot␣grad'&r,(size-s)/2,(size-s)/2,s,s,-2,0));
end
```

The output is as follows:

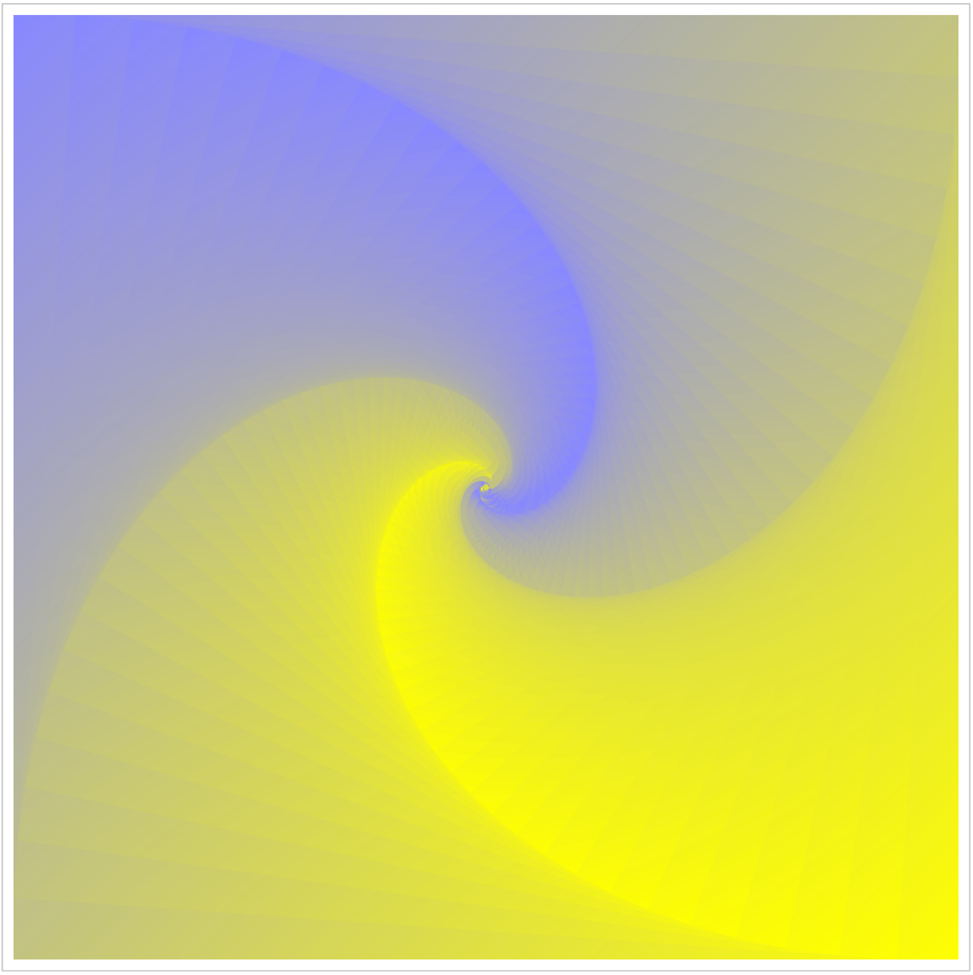


Figure 38: Colored Spiral

106 A XY-plot (xDrawXY)

–– Run LPL Code , HTML Document –

Problem:

Listing 107: The Complete Model implemented in LPL [2]

```
model xDrawXY " A XY−plot";
  Draw.Scale(3,3);
  Draw.Rect(0,0,155,155,2);
  Draw.Line(5,5,5,150,0);
  Draw.Line(5,5,150,5,0);
  Draw.Line(5,75,75,5,0);
  Draw.Line('ff',5,150,50,5,0);
  Draw.Text('A',1,4);
  Draw.Text('B',1,75);
  Draw.Text('C',40,42);
  Draw.Text('D',51,4);
  Draw.Text('y',1,150);
  Draw.Text('x',145,4);
  Draw.Circle(38.5,41.5,1.5);
  Draw.Text('6x+2y␣=␣300',27,100,-75,16);
  Draw.Text('5x+5y␣=␣350',50,35,15,0,0,1,-2,-2,-45,-45);    // x,y,h,c1,
      c2,w,o1,o2,a,r
end
```

Output as follows:

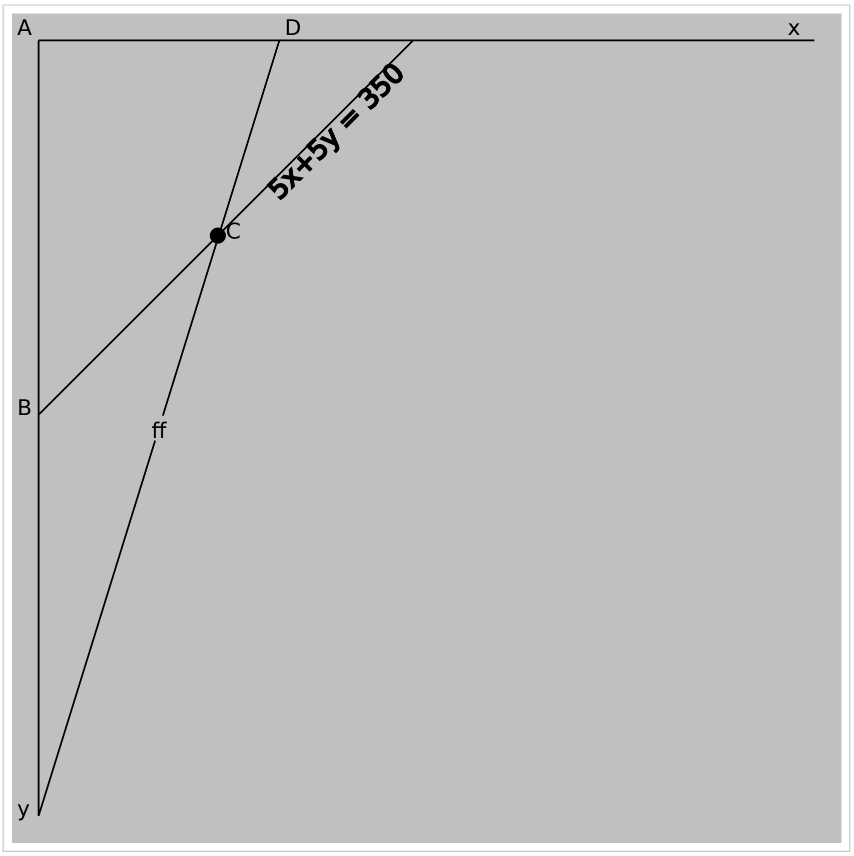


Figure 39: Output

107 A XY-Function-plot (xDrawXY1)

—– Run LPL Code , HTML Document –

Problem:

Listing 108: The Complete Model implemented in LPL [2]

```
model CHAIN " A XY−Function−plot";
  parameter n := 500;
  set i := 0..n  "number of intervals for the discretization";
  parameter
    xa := −3.14   "left x coordinate";
    xb := 6.28    "right x coordinate";
    dx   := (xb−xa)/n  "interval";
    x{i} := xa+dx*i    "x−coors";
    y{i} := Sin(x);
    z{i} := Cos(x);
    w{i} := Sin(x^2);
  Draw.DefFont('Verdana',8);
  Draw.XY(x,y,z,w,1);
end
```

The output is as follows:

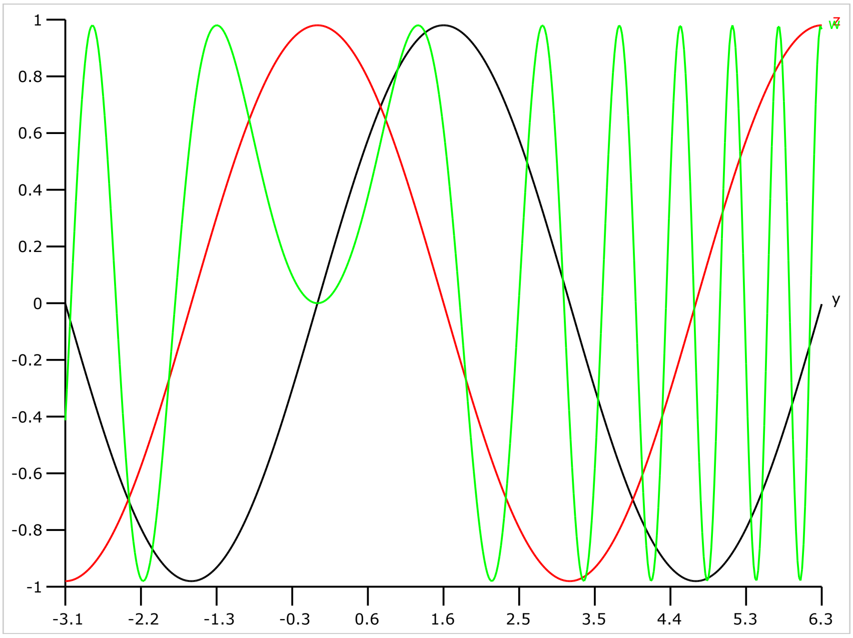


Figure 40: XY-Function-Graph

References

[1] MatMod. Homepage for Learning Mathematical Modeling : https://matmod.ch.

[2] Hürlimann T. Reference Manual for the LPL Modeling Language, most recent version. https://matmod.ch/lpl/doc/manual.pdf.