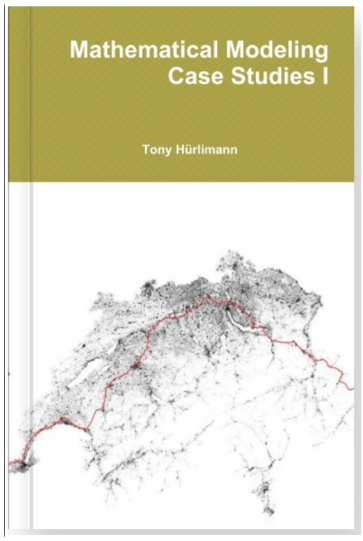

Mathematical Modeling: Case Studies I

by Tony Hürlimann , 2022, 3nd Ed., 279 pages

Hardcover book (english) to buy at Lulu.com

eBook (PDF) (english) at MatMod Shop

An Excerpt: Shortest Path

This book is a comprehensive guide to mathematical modeling. It uses classical models from the operations research as "case studies" and represents them as mathematical models. The models are coded in the computer language **LPL** and most can be solved directly over the Internet.

The book contains many modeling tricks and teachers in Mathematics on high school will be amazed how playful and diverting mathematical modeling can be. The book can also be used for an introduction course in operations research at university level.

The goal is to encourage the use of computer tools in an educational context in order to improve and develop mathematical modelling skills. It can be used as a self-study or in classes.

About the author:

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